$$= 3 \times 4.187 \ln \frac{0.67 \times 0.835 \times 0.33}{0.835^{0.33}} = 12.561 \ln \frac{0.94555}{0.94223} = 0.0442 \text{ kW/K}$$

Rate of exergy loss due to mixing

$$\dot{I} = T_0 \dot{S}_{\text{gen}} = 300 \times 0.0442 = 13.26 \text{ kW}$$

Alternatively,

Equilibrium temperature after mixing,

$$t = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2} = \frac{2 \times 90 + 1 \times 30}{2 + 1} = 70°\text{C}$$

$$\Delta \dot{S}_{\text{univ}} = \dot{S}_{\text{gen}} = 2 \times 4.187 \ln \frac{343}{363} + 1 \times 4.187 \ln \frac{343}{303}$$

$$= 0.0447 \text{ kW/K}$$

$$\therefore \quad \dot{I} = 300 \times 0.0447 = 13.41 \text{ kW}$$

Example 8.12

By burning a fuel the rate of heat release is 500 kW at 2000 K. What would be the first law and the second law efficiencies if (a) energy is absorbed in a metallurgical furnace at the rate of 480 kW at 1000 K, (b) energy is absorbed at the rate of 450 kW for generation of steam at 500 K, and (c) energy is absorbed in a chemical process at the rate of 300 kW at 320 K? Take $T_0 = 300$ K. (d) Had the energy absorption rate been equal to 450 kW in all these three cases, what would have been the second law efficiencies? What is the inference that you can draw from this example?

Solution If $\dot{Q}_r$ is the rate of heat release at temperature T_r and $\dot{Q}_a$ the rate of heat absorption at temperature T_a, then

$$\eta_I = \frac{\dot{Q}_a}{\dot{Q}_r} \quad \text{and} \quad \eta_{II} = \eta_I \frac{1 - \frac{T_0}{T_a}}{1 - \frac{T_0}{T_r}}$$

(a) Metallurgical furnace

$$\eta_I = \frac{480}{500} \times 100 = 96\%$$

$$\eta_{II} = 0.96 \frac{1 - \frac{300}{1000}}{1 - \frac{300}{2000}} \times 100 = 79\%$$

(b) Steam generation

$$\eta_I = \frac{450}{500} \times 100 = 90\%$$

$$\eta_{II} = 0.90 \frac{1 - \frac{300}{500}}{1 - \frac{300}{2000}} \times 100 = 42.3\%$$

(c) Chemical process

$$\eta_I = \frac{300}{500} \times 100 = 60\%$$

$$\eta_{II} = 0.60 \frac{1 - \frac{300}{320}}{1 - \frac{300}{2000}} \times 100 = 4.41\%$$

(d) In all the three cases, η_I would remain the same, where

$$\eta_I = \frac{450}{500} \times 100 = 0.90$$

$$\eta_{II\,(a)} = 0.90 \times \frac{1 - \frac{300}{1000}}{1 - \frac{300}{2000}} \times 100 = 74.11\%$$

$$\eta_{II\,(b)} = 0.90 \times \frac{1 - \frac{300}{500}}{1 - \frac{300}{2000}} \times 100 = 42.3\%$$

$$\eta_{II\,(c)} = 0.90 \times \frac{1 - \frac{300}{320}}{1 - \frac{300}{2000}} \times 100 = 6.61\%$$

It is seen that as the energy loss ($\dot{Q}_r - \dot{Q}_a$) increases, the first law efficiency decreases. For the same heat loss, however, as the temperature difference between the source and the use temperature increases, the second law efficiency decreases, or in other words, the rate of exergy loss increases.

Example 8.13

A system undergoes a power cycle while receiving energy Q_1 at temperature T_1 and discharging energy Q_2 at temperature T_2. There are no other heat transfers.

(*a*) *Show that the thermal efficiency of the cycle can be expressed as:*

$$\eta = 1 - \frac{T_2}{T_1} - \frac{T_2 I}{T_0 Q_1}$$

where T_0 is the ambient temperature and I is the irreversibility of the cycle.

(*b*) *Obtain an expression for the maximum theoretical value for the thermal efficiency.*

(*c*) *Derive an expression for the irreversibility for which no network is developed by the cycle. What conclusion do you derive from it?*

Solution An availability balance for the cycle gives (Fig. Ex. 8.13)

$$(\Delta A)_{\text{cycle}} = 0 = \left(1 - \frac{T_0}{T_1}\right) Q_1 - \left(1 - \frac{T_0}{T_2}\right) Q_2 - W - I$$

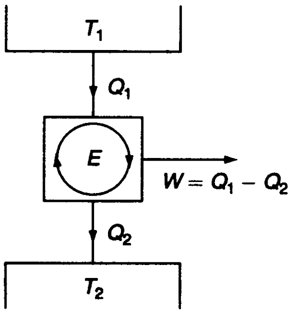
since each property is restored to its initial state.

Since $Q_2 = Q_1 - W$,

$$0 = \left(1 - \frac{T_0}{T_1}\right) Q_1 - \left(1 - \frac{T_0}{T_2}\right) (Q_1 - W) - W - I$$

$$W = \frac{T_2}{T_0}\left[\left(1 - \frac{T_0}{T_1}\right) - \left(1 - \frac{T_0}{T_2}\right)\right] Q_1 - \frac{T_2 I}{T_0}$$

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} - \frac{T_2 I}{T_0 Q_1} \qquad \textit{Proved.}$$



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(b) When $I = 0$, $\eta_{\max} = 1 - \dfrac{T_2}{T_1}$

(c) When $W = 0$ $\quad \eta = 0 = 1 - \dfrac{T_2}{T_1} - \dfrac{T_2 I}{T_0 Q_1}$

$$I = T_0\left[\frac{1}{T_2} - \frac{1}{T_1}\right] Q_1 = T_0\left[\frac{Q_1}{T_2} - \frac{Q_1}{T_1}\right] = T_0 S_{\text{gen}}$$

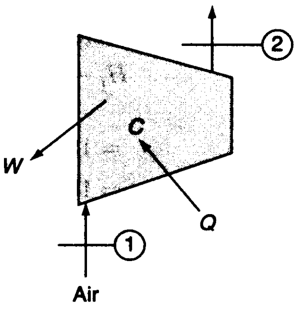
The heat transfer Q_1 from T_1 to T_2 takes place through a reversible engine, and the entire work is dissipated in the brake, from which an equal amount of heat is rejected to the reservoir at T_2. Heat transfer through a finite temperature difference is thus equivalent to the destruction of its exergy. (See Art. 8.10.1(a)).

Example 8.14

A compressor operating at steady state takes in 1 kg/s of air at 1 bar and 25°C and compresses it to 8 bar and 160°C. Heat transfer from the compressor to its surroundings occurs at a rate of 100 kW. (a) Determine the power input in kW. (b) Evaluate the second law efficiency for the compressor. Neglect KE and PE changes. Take T_0 = 25°C and P_0 = 1 bar.

Solution SFEE for the compressor (Fig. 8.14) gives:

$$\dot{W} = \dot{Q} + \dot{m}(h_1 - h_2) = -100 + 1 \times 1.005\,(25 - 160)$$
$$= -235.7\ \text{kW} \qquad \textit{Ans.}\ \text{(a)}$$



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Exergy balance for the compressor gives:

$$\dot{m} a_{f_1} + \dot{Q}\left(1 - \frac{T_0}{T}\right) - \dot{W} - \dot{m} a_{f_2} = \dot{I}$$

$$-\dot{W} = \dot{m}\,(a_{f_2} - a_{f_1}) - \dot{Q}\left(1 - \frac{T_0}{T}\right) + \dot{I}$$

$$\eta_{\text{II}} = \frac{\dot{m}\,(a_{f_2} - a_{f_1})}{\dot{W}}$$

$$a_{f_2} - a_{f_1} = h_2 - h_1 - T_0(s_2 - s_1)$$

$$= c_p(T_2 - T_1) - T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right) = 1.005\,(160 - 25) - 298\left(1.005\ln\frac{433}{298} - 0.287\ln 8\right)$$

$$= 200.95 \text{ kJ/kg}$$

$$\eta_{II} = \frac{200.95}{235.7} = 0.853 \text{ or, } 85.3\%$$ *Ans.* (b)

Example 8.15

Determine the exergy of 1 m^3 *of complete vacuum.*

Solution $\phi = U - U_0 + p_0(V - V_0) - T_0(S - S_0) = H - H_0 - V(p - p_0) - T_0(S - S_0)$

Since a vacuum has zero mass,

$$U = 0, H_0 = 0, \text{ and } S = 0$$

If the vacuum was reduced to the dead state

$$U_0 = 0, H_0 = 0, S_0 = 0 \text{ and } V_0 = 0$$

The pressure p for the vacuum is zero.

But $p_0 = 1$ bar $= 100$ kPa and $V = 1\text{ m}^3$ $\phi = p_0V = 100\,\frac{\text{kN}}{\text{m}^2} \times 1\text{ m}^3 = 100$ kJ *Ans.*

If an air motor operates between the atmosphere and the vacuum, this is the maximum useful work obtainable. Therefore, the vacuum has an exergy or work potential.

Example 8.16

A mass of 1000 *kg of fish initially at* 1 *bar,* 300 *K is to be cooled to* – 20°C. *The freezing point of fish is* – 2.2°C, *and the specific heats of fish below and above the freezing point are* 1.7 *and* 3.2 *kJ/kg K respectively. The latent heat of fusion for the fish can be taken as* 235 *kJ/kg. Calculate the exergy produced in the chilling process. Take* $T_0 = 300$ *K and* $p_0 = 1$ *bar.*

Solution Exergy produced $= H_2 - H_1 - T_0(S_2 - S_1)$

With reference to Fig. Ex. 8.16,

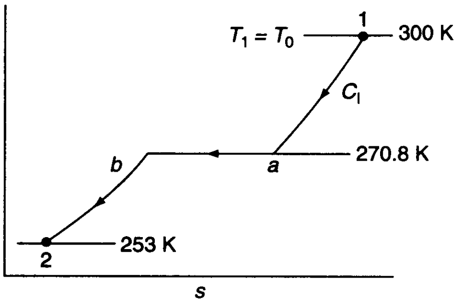
$$H_1 - H_2 = 1000\,[1.7\,(270.8 - 253) + 235 + 3.2\,(300 - 270.8)]$$

$$= 1000\,[1.7 \times 17.8 + 235 + 3.2 \times 29.2]$$

$$= 1000\,[30.26 + 235 + 93.44] = 358.7 \text{ MJ}$$

$$H_2 - H_1 = -\,358.7 \text{ MJ}$$

$$S_1 - S_2 = 1000\left[1.7\ln\frac{270.8}{253} + \frac{235}{270.8} + 3.2\ln\frac{300}{270.8}\right]$$



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$$= -\,1000\,[0.1156 + 0.8678 + 0.3277] = 1.311 \text{ MJ/K}$$

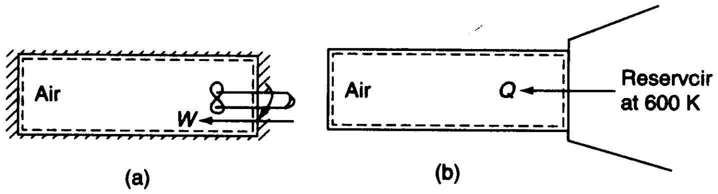
$$S_2 - S_1 = -\,1.311 \text{ MJ/K}$$

Exergy produced $= -\,358.7 + 300 \times 1.311 = -\,358.7 + 393.3 = 34.6$ MJ or 9.54 kWh *Ans.*

Example 8.17

A quantity of air initially at 1 bar, 300 K undergoes two types of interactions: (a) it is brought to a final temperature of 500 K adiabatically by paddle-wheel work transfer, (b) the same temperature rise is brought about by heat transfer from a thermal reservoir at 600 K. Take $T_0 = 300$ K, $p_0 = 1$ atm. Determine the irreversibility (in kJ/kg) in each case and comment on the results.

Solution *Case (a)*: As shown in the above Figures (Fig. Ex. 8.17),



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$$\Delta s_{\text{univ}} = s_{\text{gen}} = c_v \ln \frac{T_2}{T_1} = 0.718 \ln \frac{500}{300} = 0.367 \text{ kJ/kg K}$$

$$I = 300 \times 0.367 = 110.1 \text{ kJ/kg} \qquad \textit{Ans.}$$

Case (b):

$$Q = m\, c_v (T_2 - T_1)$$

$$= 1 \times 0.718\,(500 - 300) = 143.6 \text{ kJ/kg}$$

$$\Delta s_{\text{univ}} = s_2 - s_1 - \frac{Q_2}{T} = 0.367 - \frac{143.6}{600}$$

$$= 0.1277 \text{ kJ/kg K} I = 300 \times 0.1277 = 38.31 \text{ kJ/kg}$$

Comment:

The irreversibility in case (b) is less than in case (a).

$$I_a = T_0(s_2 - s_1),\ I_b = T_0(s_2 - s_1) - \frac{Q}{T}$$

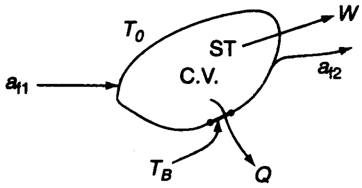
$$I_a - I_b = \frac{Q}{T}$$

The irreversibility in case (b) is always less than in case (a) and the two values would approach each other only at high reservoir temperature, i.e.,

$$I_a \rightarrow I_b \text{ as } T \rightarrow \infty$$

Example 8.18

Steam enters a turbine at 30 bar, 400°C (h = 3230 kJ/kg, s = 6.9212 kJ/kg K) and with a velocity of 160 m/s. Steam leaves as saturated vapour at 100°C (h = 2676.1 kJ/kg, s = 7.3549 kJ/kg K) with a velocity of 100 m/s. At steady state the turbine develops work at a rate of 540 kJ/kg. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 500 K. Determine the irreversibility per unit mass. Give an exergy balance and estimate the second law efficiency of the turbine. Take $p_0 = 1$ atm, $T_0 = 298$ K and neglect PE effect.



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Solution By exergy balance of the control volume (Fig. Ex. 8.18),

$$a_{f_1} = W + Q\left(1 - \frac{T_0}{T_B}\right) + a_{f_2} + I$$

where a_f is the exergy transfer per unit mass.

$$I = a_{f_1} - a_{f_2} - W - Q\left(1 - \frac{T_0}{T_B}\right)$$

$$= (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} - W - Q\left(1 - \frac{T_0}{T_B}\right)$$

$$= (3230.9 - 2676.1) - 298(6.9212 - 7.3549) + \frac{160^2 - 100^2}{2}$$

$$\times 10^{-3} - 540 - Q\left(1 - \frac{298}{500}\right)$$

$$= 151.84 - Q(0.404) \qquad (1)$$

By SFEE,

$$h_1 + \frac{V_1^2}{2} = W + h_2 + Q + \frac{V_2^2}{2}$$

$$Q = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} - W$$

$$= (3230.9 - 2676.1) + \frac{160^2 - 100^2}{2} \times 10^{-3} - 540$$

$$= 22.6 \text{ kJ/kg.}$$

From Eq. (1),

$$I = 151.84 - 22.6 \times 0.404$$

$$= 142.71 \text{ kJ/kg}$$ *Ans.*

Net exergy transferred to turbine

$$a_{f_1} - a_{f_2} = 691.84 \text{ kJ/kg}$$

Work = 540 kJ/kg

Exergy destroyed = I = 142.71 kJ/kg

Exergy transferred out accompanying heat transfer

$$= 22.6 \times 0.404 = 9.13 \text{ kJ/kg}$$

Exergy Balance

Exergy transferred	*Exergy utilized*
691.84 kJ/kg	Work = 540 kJ/kg (78%) Destroyed = 142.71 kJ/kg (20.6%) Transferred with heat = 9.13 kJ/kg (1.3%)
	691.84 kJ/kg

Second law efficiency, $\eta_{II} = \dfrac{540}{691.84} = 0.78$ or 78% *Ans.*

Example 8.19

A furnace is heated by an electrical resistor. At steady state, electrical power is supplied to the resistor at a rate of 8.5 kW per metre length to maintain it at 1500 K when the furnace walls are at 500 K. Let T_0 = 300 K (a) For the resistor as the system, determine the rate of availability transfer accompanying heat and the irreversibility rate, (b) For the space between the resistor and the walls as the system, evaluate the irreversibility rate.

Solution *Case* (*a*): At steady state for the resistor (Fig. Ex. 8.19).

$$\dot{Q} = \Delta\dot{U} + \dot{W} = \dot{W} = 8.5 \text{ kW}$$

Availability rate balance gives

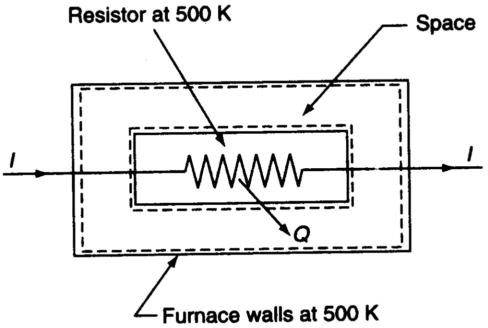
$$\frac{\mathrm{d}A}{\mathrm{d}t} = \left(1 - \frac{T_0}{T}\right)\dot{Q} + \left(\dot{W} - p_0\frac{\mathrm{d}V}{\mathrm{d}t}\right) - \dot{I} = 0$$

$\dot{I}$ = Rate of irreversibility

$$= \left(1 - \frac{T_0}{T}\right)\dot{Q} + \dot{W} = \left(1 - \frac{300}{1500}\right)(-8.5) + 8.5$$

$$= 1.7 \text{ kW}$$

Ans. (a)



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Rate of availability transfer with heat

$$= \left(1 - \frac{T_0}{T}\right)\dot{Q} = \left(1 - \frac{300}{1500}\right)(-8.5) = -6.8 \text{ kW}$$

Ans. (a)

Case (*b*): Steady state,

$$\frac{\mathrm{d}A}{\mathrm{d}T} = \left(1 - \frac{T_0}{T}\right)\dot{Q} - \left(1 - \frac{T_0}{T_w}\right)\dot{Q} - \dot{W} - \dot{I} = 0$$

$$\dot{I} = \left(1 - \frac{300}{1500}\right)8.5 - \left(1 - \frac{300}{500}\right)8.5$$

$$= 6.8 - 3.4 = 3.4 \text{ kW}$$

Ans. (b)

Example 8.20

Air enters a compressor at 1 bar, 30°C, which is also the state of the environment. It leaves at 3.5 bar, 141°C and 90 m/s. Neglecting inlet velocity and P.E. effect, determine (a) whether the compression is adiabatic or polytropic, (b) if not adiabatic, the polytropic index, (c) the isothermal efficiency, (d) the minimum work input and irreversibility, and (d) the second law efficiency. Take c_p of air = 1.0035 kJ/kg K.

Solution (a) After isentropic compression

$$\frac{T_{2s}}{T_1} = \left[\frac{p_2}{p_1}\right]^{(\gamma - 1)/\gamma}$$

$$T_{2s} = 303\ (3.5)^{0.286} = 433.6 \text{ K} = 160.6°\text{C}$$

Since this temperature is higher than the given temperature of 141°C, there is heat loss to the surroundings. The compression cannot be adiabatic. It must be polytropic.

(b)

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(n-1)/n}$$

$$\frac{141+273}{30+273} = 1.366 = \left[\frac{3.5}{1}\right]^{(n-1)/n}$$

$$\log 1.366 = \frac{n-1}{n} \log 3.5$$

$$1 - \frac{1}{n} = \frac{0.135}{0.544} = 0.248$$

$$n = 1.32978 = 1.33$$

Ans.

(c) Actual work of compression

$$W_a = h_1 - h_2 - \frac{V_2^2}{2} = 1.0035\,(30 - 141) - \frac{90^2}{2} \times 10^{-3} = -\,115.7 \text{ kJ/kg}$$

Isothermal work $W_T = \int_1^2 v\,dp - \frac{V_2^2}{2} = -\,RT_1 \ln \frac{p_2}{p_1} - \frac{V_2^2}{2}$

$$= -\,0.287 \times 303 \ln (3.5) - \frac{90^2}{2} \times 10^{-3} = -\,113 \text{ kJ/kg}$$

Isothermal efficiency:

$$\eta_T = \frac{W_T}{W_a} = \frac{113}{115.7} = 0.977 \text{ or } 97.7\%$$

Ans.

(d) Decrease in availability or exergy:

$$\Psi_1 - \Psi_2 = h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}$$

$$= c_p(T_1 - T_2) - T_0 \left[R \ln \frac{p_2}{p_1} - c_p \ln \frac{T_2}{T_1}\right] - \frac{V_2^2}{2}$$

$$= 1.0035\,(30 - 141)$$

$$-\,303 \left[0.287 \ln 3.5 - 1.0035 \ln \frac{414}{303}\right] - \frac{90^2}{2000}$$

$$= -\,101.8 \text{ kJ/kg}$$

Minimum work input = – 101.8 kJ/kg

Ans.

Irreversibility, $I = W_{rev} - W_a$

$$= -\,101.8 - (-\,115.7)$$

$$= 13.9 \text{ kJ/kg}$$

Ans.

(e) Second law efficiency,

$$\eta_{II} = \frac{\text{Minimum work input}}{\text{Actual work input}} = \frac{101.8}{115.7}$$

$$= 0.88 \text{ or } 88\%$$

Ans.

Review Questions

8.1 What do you understand by high grade energy and low grade energy?

8.2 What is available energy and unavailable energy?

8.3 Who propounded the concept of availability?

8.4 What is the available energy referred to a cycle?

8.5 Show that there is a decrease in available energy when heat is transferred through a finite temperature difference.

8.6 Deduce the expression for available energy from a finite energy source at temperature T when the environmental temperature is T_0.

8.7 What do you understand by exergy and energy?

8.8 What is meant by quality of energy?

8.9 Why is exergy of a fluid at a higher temperature more than that at a lower temperature?

8.10 How does the exergy value provide a useful measure of the quality of energy?

8.11 Why is the second law called the law of degradation of energy?

8.12 Energy is always conserved, but its quality is always degraded. Explain.

8.13 Why is the work done by a closed system in a reversible process by interacting only with the surroundings the maximum?

8.14 Show that equal work is done in all reversible processes between the same end states of a system if it exchanges energy only with the surroundings.

8.15 Give the general expression for the maximum work of an open system which exchanges heat only with the surroundings.

8.16 What do you understand by Keenan function?

8.17 Give the expression for reversible work in a steady flow process under a given environment.

8.18 Give the expression for reversible work done by a closed system if it interacts only with the surroundings.

8.19 What do you understand by 'useful work'? Derive expressions for useful work for a closed system and a steady flow system which interact only with the surroundings.

8.20 What are the availability functions for a: (a) closed system, (b) steady flow system?

8.21 What do you understand by the dead state?

8.22 What is meant by availability?

8.23 Give expressions for availabilities of a closed system and a steady flow open system.

8.24 What are Helmholtz function and Gibbs function?

8.25 What is the availability in a chemical reaction if the temperature before and after the reaction is the same and equal to the temperature of the surroundings?

8.26 When is the availability of a chemical reaction equal to the decrease in the Gibbs function?

8.27 Derive the expression for irreversibility or exergy loss in a process executed by: (a) a closed system, (b) a steady flow system, in a given environment.

8.28 State and explain the Gouy-Stodola theorem.

8.29 How is heat transfer through a finite temperature difference equivalent to the destruction of its availability?

8.30 Considering the steady and adiabatic flow of an ideal gas through a pipe, show that the rate of decrease in availability or lost work is proportional to the pressure drop and the mass flow rate.

8.31 What do you understand by Grassman diagram?

8.32 What is entropy generation number?

8.33 Why is exergy always a positive value? Why cannot it be negative?

8.34 Why and when is exergy completely destroyed?

8.35 Give the exergy balance for a closed system.

8.36 Explain the statement: The exergy of an isolated system can never increase. How is it related to the principle of increase of entropy?

8.37 Give the exergy balance of a steady flow system.

8.38 Derive expressions for the irreversibility and second law efficiency of a:

(i) steam turbine (ii) compressor

(iii) heat exchanger, and (iv) mixer

8.39 What is the deficiency of the first law efficiency? How does the second law efficiency make up this deficiency?

8.40 How can you improve the first law efficiency and the second law efficiency?

Problems

8.1 What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if (a) a temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand, and the heat engine and the heat sink, on the other, (b) the source temperature drops by 50°C and the sink temperature rises by 50°C during the heat transfer process according to the linear law $\frac{đQ}{dT} = \pm$ constant?

Ans. 57.38 kJ, (a) 11.46 kJ, (b) 5.5 kJ

8.2 In a steam generator, water is evaporated at 260°C, while the combustion gas ($c_p = 1.08$ kJ/kg K) is cooled from 1300°C to 320°C. The surroundings are at 30°C. Determine the loss in available energy due to the above heat transfer per kg of water evaporated. (Latent heat of vaporization of water at 260°C = 1662.5 kJ/kg.)

Ans. 443.6 kJ

8.3 Exhaust gases leave an internal combustion engine at 800°C and 1 atm, after having done 1050 kJ of work per kg of gas in the engine (c_p of gas = 1.1 kJ/kg K). The temperature of the surroundings is 30°C. (a) How much available energy per kg of gas is lost by throwing away the exhaust gases? (b) What is the ratio of the lost available energy to the engine work? *Ans.* (a) 425.58 kJ, (b) 0.405

8.4 A hot spring produces water at a temperature of 56°C. The water flows into a large lake, with a mean temperature of 14°C, at a rate of 0.1 m^3 of water per min. What is the rate of working of an ideal heat engine which uses all the available energy? *Ans.* 19.5 kW

8.5 0.2 kg of air at 300°C is heated reversibly at constant pressure to 2066 K. Find the available and unavailable energies of the heat added. Take T_0 = 30°C and c_p = 1.0047 kJ/kg K.

Ans. 211.9 and 78.1 kJ

8.6 Eighty kg of water at 100°C are mixed with 50 kg of water at 60°C, while the temperature of the surroundings is 15°C. Determine the decrease in available energy due to mixing. *Ans.* 240 kJ

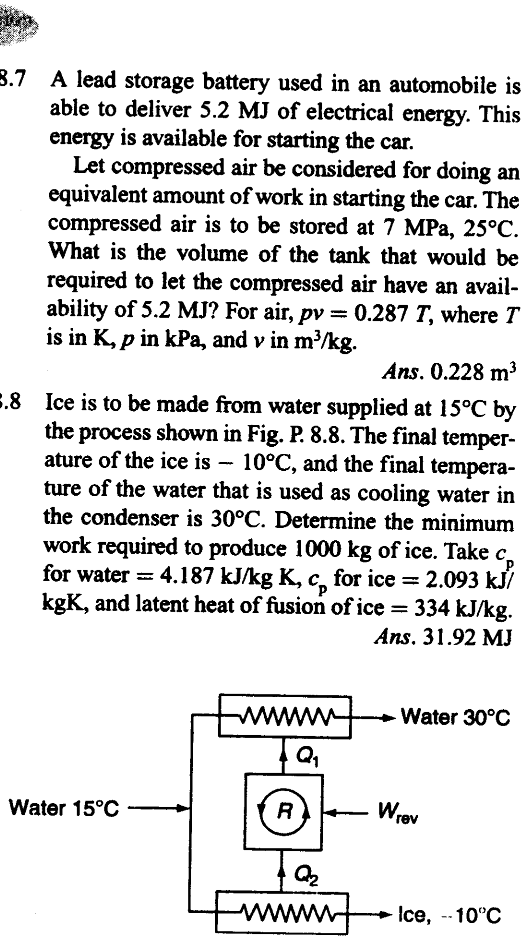
8.7 A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is available for starting the car.

Let compressed air be considered for doing an equivalent amount of work in starting the car. The compressed air is to be stored at 7 MPa, 25°C. What is the volume of the tank that would be required to let the compressed air have an availability of 5.2 MJ? For air, $pv = 0.287\,T$, where T is in K, p in kPa, and v in m^3/kg.

Ans. 0.228 m^3

8.8 Ice is to be made from water supplied at 15°C by the process shown in Fig. P. 8.8. The final temperature of the ice is − 10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C. Determine the minimum work required to produce 1000 kg of ice. Take c_p for water = 4.187 kJ/kg K, c_p for ice = 2.093 kJ/kgK, and latent heat of fusion of ice = 334 kJ/kg.

Ans. 31.92 MJ



[illegible]

8.9 A pressure vessel has a volume of 1 m^3 and contains air at 1.4 MPa, 175°C. The air is cooled to 25°C by heat transfer to the surroundings at 25°C. Calculate the availability in the initial and final states and the irreversibility of this process. Take p_0 = 100 kPa.

Ans. 135 kJ/kg, 114.6 kJ/kg, 222 kJ

8.10 Air flows through an adiabatic compressor at 2 kg/s. The inlet conditions are 1 bar and 310 K and the exit conditions are 7 bar and 560 K. Compute the net rate of availability transfer and the irreversibility. Take T_0 = 298 K.

Ans. 481.1 kW and 21.2 kW

8.11 An adiabatic turbine receives a gas (c_p = 1.09 and c_v = 0.838 kJ/kg K) at 7 bar and 1000°C and discharges at 1.5 bar and 665°C. Determine the second law and isentropic efficiencies of the turbine. Take T_0 = 298 K.

Ans. 0.956, 0.879

8.12 Air enters an adiabatic compressor at atmospheric conditions of 1 bar, 15°C and leaves at 5.5 bar. The mass flow rate is 0.01 kg/s and the efficiency of the compressor is 75%. After leaving the compressor, the air is cooled to 40°C in an aftercooler. Calculate (a) the power required to drive the compressor, and (b) the rate of irreversibility for the overall process (compressor and cooler).

Ans. (a) 2.42 kW, (b) 1 kW

8.13 In a rotary compressor, air enters at 1.1 bar, 21°C where it is compressed adiabatically to 6.6 bar, 250°C. Calculate the irreversibility and the entropy production for unit mass flow rate. The atmosphere is at 1.03 bar, 20°C. Neglect the K.E. changes. *Ans.* 19 kJ/kg, 0.064 kJ/kg K

8.14 In a steam boiler, the hot gases from a fire transfer heat to water which vaporizes at a constant temperature of 242.6°C (3.5 MPa). The gases are cooled from 1100 to 430°C and have an average specific heat, c_p = 1.046 kJ/kg K over this temperature range. The latent heat of vaporization of steam at 3.5 MPa is 1753.7 kJ/kg. If the steam generation rate is 12.6 kg/s and there is negligible heat loss from the boiler, calculate: (a) the rate of heat transfer, (b) the rate of loss of exergy of the gas, (c) the rate of gain of exergy of the steam, and (d) the rate of entropy generation. Take T_0 = 21°C.

Ans. (a) 22096 kW, (b) 15605.4 kW
(c) 9501.0 kW, (d) 20.76 kW/K

8.15 An economizer, a gas-to-water finned tube heat exchanger, receives 67.5 kg/s of gas, c_p = 1.0046 kJ/kg K, and 51.1 kg/s of water, c_p = 4.186 kJ/kg. The water rises in temperature from 402 to 469 K, where the gas falls in temperature from 682 to 470 K. There are no changes of kinetic energy, and p_0 = 1.03 bar and T_0 = 289 K. Determine: (a) rate of change of availability of the water, (b) the rate of change of availability of the gas, and (c) the rate of entropy generation. *Ans.* (a) 4802.2 kW,
(b) 7079.8 kW, (c) 7.73 kW/K

8.16 The exhaust gases from a gas turbine are used to heat water in an adiabatic counterflow heat exchanger. The gases are cooled from 260 to 120°C, while water enters at 65°C. The flow rates of the gas and water are 0.38 kg/s and 0.50 kg/s respectively. The constant pressure specific heats for the gas and water are 1.09 and 4.186 kJ/kg K respectively. Calculate the rate of exergy loss due to heat transfer. Take T_0 = 35°C.

Ans. 11.92 kW

8.17 The exhaust from a gas turbine at 1.12 bar, 800 K flows steadily into a heat exchanger which cools the gas to 700 K without significant pressure drop. The heat transfer from the gas heats an air flow at constant pressure, which enters the heat exchanger at 470 K. The mass flow rate of air is twice that of the gas and the surroundings are at 1.03 bar, 20°C. Determine: (a) the decrease in availability of the exhaust gases, and (b) the total entropy production per kg of gas. (c) What arrangement would be necessary to make the heat transfer reversible and how much would this increase the power output of the plant per kg of turbine gas? Take c_p for exhaust gas as 1.08 and for air as 1.05 kJ/kg K. Neglect heat transfer to the surroundings and the changes in kinetic and potential energy.

Ans. (a) 66 kJ/kg, (b) 0.0731 kJ/kg K,
(c) 38.7 kJ/kg

8.18 An air preheater is used to heat up the air used for combustion by cooling the outgoing products of combustion from a furnace. The rate of flow of the products is 10 kg/s, and the products are cooled from 300°C to 200°C, and for the products at this temperature c_p = 1.09 kJ/kg K. The rate of air flow is 9 kg/s, the initial air temperature is 40°C, and for the air c_p = 1.005 kJ/kg K.

(a) What is the initial and final availability of the products?
(b) What is the irreversibility for this process?
(c) If the heat transfer from the products were to take place reversibly through heat engines, what would be the finally temperature of the air? What power would be developed by the heat engines? Take T_0 = 300 K.

Ans. (a) 85.97, 39.68 kJ/kg, (b) 256.5 kW,
(c) 394.41 K, 353.65 kW

8.19 A mass of 2 kg of air in a vessel expands from 3 bar, 70°C to 1 bar, 40°C, while receiving 1.2 kJ of heat from a reservoir at 120°C. The environment is at 0.98 bar, 27°C. Calculate the maximum work and the work done on the atmosphere.

Ans. 177 kJ, 112.5 kJ

8.20 Air enters the compressor of a gas turbine at 1 bar, 30°C and leaves the compressor at 4 bar. The compressor has an efficiency of 82%. Calculate per kg of air (a) the work of compression, (b) the reversible work of compression, and (c) the irreversibility.

[For air, use $\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma - 1/\gamma}$

where T_{2s} is the temperature of air after isentropic compression and $\gamma = 1.4$. The compressor efficiency is defined as $(T_{2s} - T_1)/(T_2 - T_1)$, where T_2 is the actual temperature of air after compression].

Ans. (a) 180.5 kJ/kg, (b) 159.5 kJ/kg (c) 21 kJ/kg

8.21 A mass of 6.98 kg of air is in a vessel at 200 kPa, 27°C. Heat is transferred to the air from a reservoir at 727°C. until the temperature of air rises to 327°C. The environment is at 100 kPa, 17°C. Determine (a) the initial and final availability of air, (b) the maximum useful work associated with the process.

Ans. (a) 103.5, 621.9 kJ (b) 582 kJ

8.22 Air enters a compressor in steady flow at 140 kPa, 17°C and 70 m/s and leaves it at 350 kPa, 127°C and 110 m/s. The environment is at 100 kPa, 7°C. Calculate per kg of air (a) the actual amount of work required, (b) the minimum work required, and (c) the irreversibility of the process.

Ans. (a) 114.4 kJ, (b) 97.3 kJ, (c) 17.1 kJ

8.23 Air expands in a turbine adiabatically from 500 kPa, 400 K and 150 m/s to 100 kPa, 300 K and 70 m/s. The environment is at 100 kPa, 17°C. Calculate per kg of air (a) the maximum work output, (b) the actual work output, and (c) the irreversibility.

Ans. (a) 159 kJ, (b) 109 kJ, (c) 50 kJ

8.24 Calculate the specific exergy of air for a state at 2 bar, 393.15 K when the surroundings are at 1 bar, 293.15 K. Take $c_p = 1$ and $R = 0.287$ kJ/kg K.

Ans. 72.31 kJ/kg

8.25 Calculate the specific exergy of CO_2 ($c_p = 0.8659$ and $R = 0.1889$ kJ/kg K) for a state at 0.7 bar, 268.15 K and for the environment at 1.0 bar and 293.15 K. *Ans.* – 18.77 kJ/kg

8.26 A pipe carries a stream of brine with a mass flow rate of 5 kg/s. Because of poor thermal insulation the brine temperature increases from 250 K at the pipe inlet to 253 K at the exit. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) associated with the heat leakage. Take $T_0 = 293$ K and $c_p = 2.85$ kJ/kg K. *Ans.* 7.05 kW

8.27 In an adiabatic throttling process, energy per unit mass of enthalpy remains the same. However, there is a loss of exergy. An ideal gas flowing at the rate $\dot{m}$ is throttled from pressure p_1 to pressure p_2 when the environment is at temperature T_0. What is the rate of exergy loss due to throttling?

Ans. $\dot{I} = \dot{m} R T_0 \ln \frac{p_1}{p_2}$

8.28 Air at 5 bar and 20°C flows into an evacuated tank until the pressure in the tank is 5 bar. Assume that the process is adiabatic and the temperature of the surroundings is 20°C. (a) What is the final temperature of the air? (b) What is the reversible work produced between the initial and final states of the air? (c) What is the net entropy change of the air entering the tank? (d) Calculate the irreversibility of the process.

Ans. (a) 410.2 K, (b) 98.9 kJ/kg,
(c) 0.3376 kJ/kg K, (d) 98.9 kJ/kg

8.29 A Carnot cycle engine receives and rejects heat with a 20°C temperature differential between itself and the thermal energy reservoirs. The expansion and compression processes have a pressure ratio of 50. For 1 kg of air as the working substance, cycle temperature limits of 1000 K and 300 K and $T_0 = 280$ K, determine the second law efficiency. *Ans.* 0.965

8.30 Energy is received by a solar collector at the rate of 300 kW from a source temperature of 2400 K. If 60 kW of this energy is lost to the surroundings at steady state and if the user temperature remains constant at 600 K, what are the first law and the second law efficiencies? Take $T_0 = 300$ K.

Ans. 0.80, 0.457

8.31 For flow of an ideal gas through an insulated pipeline, the pressure drops from 100 bar to 95 bar. If the gas flows at the rate of 1.5 kg/s and has $c_p = 1.005$ and $c_v = 0.718$ kJ/kg K and if $T_0 = 300$ K, find the rate of entropy generation, and rate of loss of exergy.

Ans. 0.0215 kW/K, 6.46 kW

8.32 The cylinder of an internal combustion engine contains gases at 2500°C, 58 bar. Expansion takes place through a volume ratio of 9 according to $pv^{1.38}$ = const. The surroundings are at 20°C, 1.1 bar. Determine the loss of availability, the work transfer and the heat transfer per unit mass. Treat the gases as ideal having $R = 0.26$ kJ/kg K and $c_v = 0.82$ kJ/kg K.

Ans. 1144 kJ/kg, 1074 kJ/kg, – 213 kJ/kg

8.33 In a counterflow heat exchanger, oil ($c_p = 2.1$ kJ/kg K) is cooled from 440 to 320 K, while water ($c_p = 4.2$ kJ/kg K) is heated from 290 K to temperature T. The respective mass flow rates of oil and water are 800 and 3200 kg/h. Neglecting pressure drop, KE and PE effects and heat loss, determine (a) the temperature T, (b) the rate of exergy destruction, (c) the second law efficiency. Take $T_0 = 17$°C and $p_0 = 1$ atm.

Ans. (a) 305 K, (b) 41.4 MJ/h, (c) 10.9%

8.34 Oxygen enters a nozzle operating at steady state at 3.8 MPa, 387°C and 10 m/s. At the nozzle exit the conditions are 150 kPa, 37°C and 750 m/s. Determine (a) the heat transfer per kg and (b) the irreversibility. Assume oxygen as an ideal gas, and take $T_0 = 20$°C, $p_0 = 1$ atm.

Ans. (a) – 37.06 kJ/kg, (b) 81.72 kJ/kg

8.35 Argon gas expands adiabatically in a turbine from 2 MPa, 1000°C to 350 kPa. The mass flow rate is 0.5 kg/s and the turbine develops power at the rate of 120 kW. Determine (a) the temperature of argon at the turbine exit, (b) the irreversibility rate, and (c) the second law efficiency. Neglect KE and PE effects and take $T_0 = 20$°C, $p_0 = 1$ atm.

Ans. (a) 538.1°C (b) 18.78 kW, (c) 86.5%

8.36 In the boiler of a power plant are tubes through which water flows as it is brought from 0.8 MPa, 150°C ($h = 632.6$ kJ/kg, $s = 1.8418$ kJ/kg K) to 0.8 MPa, 250°C ($h = 2950$ kJ/kg, $s = 7.0384$ kJ/kg K). Combustion gases passing over the tubes cool from 1067°C to 547°C. These gases may be considered as air (ideal gas) having $c_p = 1.005$ kJ/kg K. Assuming steady state and neglecting any heat loss, and KE and PE effects, determine (a) the mass flow rate of combustion gases per kg of steam, (b) the loss of exergy per kg steam, and (c) the second law efficiency. Take $T_0 = 25$°C, $p_0 = 1$ atm.

Ans. (a) $m_g/m_w = 4.434$, (b) 802.29 kJ/kg steam, (c) 48.9%

8.37 Air enters a hair dryer at 22°C, 1 bar with a velocity of 3.7 m/s and exits at 83°C, 1 bar with a velocity of 9.1 m/s through an area of 18.7 cm^2. Neglecting any heat loss and PE effect and taking $T_0 = 22$°C, (a) evaluate the power required in kW, and (b) devise and evaluate a second law efficiency.

Ans. (a) – 1.02 kW, (b) 9%

8.38 An isolated system consists of two solid blocks. One block has a mass of 5 kg and is initially at 300°C. The other block has a mass of 10 kg and is initially at – 50°C. The blocks are allowed to come into thermal equilibrium. Assuming the blocks are incompressible with constant specific heats of 1 and 0.4 kJ/kg K, respectively, determine (a) the final temperature, (b) the irreversibility. Take $T_0 = 300$ K. *Ans.* (a) 417.4 K, (b) 277 kJ

8.39 Air flows into a heat engine at ambient conditions 100 kPa, 300 K. Energy is supplied as 1200 kJ per kg air from a 1500 K source and in some part of the process, a heat loss of 300 kJ/kg air happens at 750 K. The air leaves the engine at 100 kPa, 800 K. Find the first and the second law efficiencies. *Ans.* 0.315, 0.672

8.40 Consider two rigid containers each of volume 1 m^3 containing air at 100 kPa, 400 K. An internally reversible Carnot heat pump is then thermally connected between them so that it heats one up and cools the other down. In order to transfer heat at a reasonable rate, the temperature difference between the working fluid inside the heat pump and the air in the containers is set to 20°C. The process stops when the air in the coldest tank reaches 300 K. Find the final temperature of the air that is heated up, the work input to the heat pump, and the overall second law efficiency.

Ans. 550 K, 31.2 kJ, 0.816

CHAPTER 9

Properties of Pure Substances

A pure substance is a substance of constant chemical composition throughout its mass. It is a one-component system. It may exist in one or more phases.

9.1 $p-v$ DIAGRAM FOR A PURE SUBSTANCE

Assume a unit mass of ice (solid water) at – 10°C and 1 atm contained in a cylinder and piston machine (Fig. 9.1). Let the ice be heated slowly so that its temperature is always uniform. The changes which occur in the mass of water would be traced as the temperature is increased while the pressure is held constant. Let the state changes of water be plotted on $p-v$ coordinates. The distinct regimes of heating, as shown in Fig. 9.2, are:

1–2 The temperature of ice increases from –10°C to 0°C. The volume of ice would increase, as would be the case for any solid upon heating. At state 2, i.e. 0°C, the ice would start melting.

2–3 Ice melts into water at a constant temperature of 0°C. At state 3, the melting process ends. *There is a decrease in volume, which is a peculiarity of water.*

3–4 The temperature of water increases, upon heating, from 0°C to 100°C. The volume of water increases because of thermal expansion.

4–5 The water starts boiling at state 4 and boiling ends at state 5. This phase change from liquid to vapour occurs at a constant temperature of 100°C (the pressure being constant at 1 atm). There is a large increase in volume.

5–6 The vapour is heated to, say, 250°C (state 6). The volume of vapour increases from v_5 to v_6.

Water existed in the solid phase between 1 and 2, in the liquid phase between 3 and 4, and in the gas phase beyond 5. Between 2 and 3, the solid changed into the liquid phase by absorbing the latent heat of fusion and between 4 and 5, the liquid changed into the vapour phase by absorbing the latent heat of vaporization, both at constant temperature and pressure.

The states 2, 3, 4 and 5 are known as *saturation states.* A saturation state is a state from which a change of phase may occur without a change of pressure or temperature. State 2 is a *saturated solid state* because a solid can change into liquid at constant pressure and temperature from state 2. States 3 and 4 are both saturated liquid states. In state 3, the liquid is saturated with respect to solidification, whereas in state 4, the liquid is saturated with respect to vaporization. State 5 is a *saturated vapour state*, because from state 5, the vapour can condense into liquid without a change of pressure or temperature.

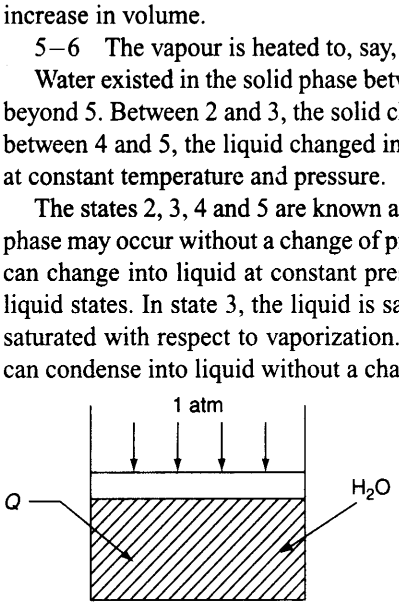


Fig. 9.1 *Heating of H_2O at a constant pressure of 1 atm*

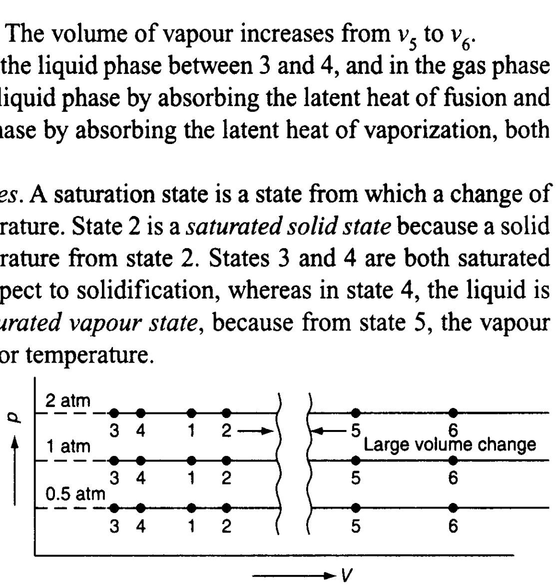


Fig. 9.2 *Changes in the volume of water during heating at constant pressure*

If the heating of ice at – 10°C to steam at 250°C were done at a constant pressure of 2 atm, similar regimes of heating would have been obtained with similar saturation states 2, 3, 4 and 5, as shown in Fig. 9.2. All the state changes of the system can similarly be plotted on the $p-v$ coordinates, when it is heated at different constant pressures. All the saturated solid states 2 at various pressures are joined by a line, as shown in Fig. 9.3.

Similarly, all the saturated liquid states 3 with respect to solidification, all the saturated liquid states 4 with respect to vaporization, and all the saturated vapour states 5, are joined together.

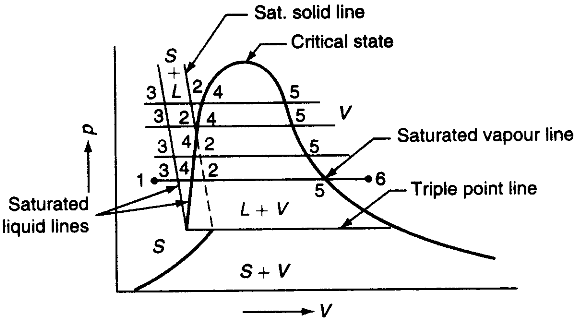


Fig. 9.3 *p–v diagram of water, whose volume decreases on melting*

Figure 9.4 shows state changes of a pure substance other than water whose volume increases on melting.

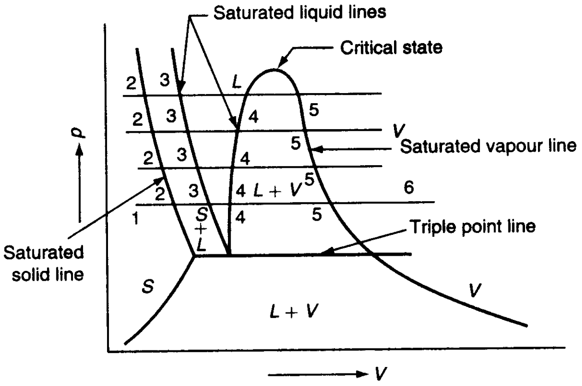


Fig. 9.4 *p–v diagram of a pure substance other than water, whose volume increases on melting*

The line passing through all the saturated solid states 2 (Figs. 9.3 and 9.4) is called the *saturated solid line*. The lines passing through all the saturated liquid states 3 and 4 with respect to solidification and vaporization respectively are known as the *saturated liquid lines*, and the line passing through all the saturated vapour states 5, is the *saturated vapour line*. The saturated liquid line with respect to vaporization and the saturated vapour line incline towards each other and form what is known as the *saturation* or *vapour dome*. The two lines meet at the *critical state*.

To the left of the saturated solid line is the *solid* (S) *region* (Fig. 9.4). Between the saturated solid line and saturated liquid line with respect to solidification there exists the *solid-liquid mixture* ($S + L$) region. Between the two saturated liquid lines is the *compressed liquid region*. The *liquid-vapour mixture region* ($L + V$) exists within the vapour dome between the saturated liquid and saturated vapour lines. to the right of the saturated vapour line is the *vapour region*. The *triple point* is a line on the $p-v$ diagram, where all the three phases, solid, liquid, and gas, exist in equilibrium. At a pressure below the triple point line, the substance cannot exist in the liquid phase, and the substance, when heated, transforms from solid to vapour (known as sublimation) by absorbing the latent heat of sublimation from the surroundings. The region below the triple point line is, therefore, the *solid-vapour* ($S + V$) *mixture region*. Table 9.1 gives the triple point data for a number of substances.

Liquid is, most often, the working fluid in power cycles, etc. and interest is often confined to the liquid-vapour regions only. So to locate the state points, the solid regions from Figs 9.3 and 9.4 can be omitted. The $p-v$ diagram then becomes as shown in Fig. 9.5. If the vapour at state A is compressed slowly and isothermally, the pressure will rise until there is saturated vapour at point B. If the compression is continued, condensation takes place, the pressure remaining constant so long as the temperature remains constant. At any point between B and C, the liquid and vapour are in equilibrium. Since a very large increase in pressure is needed to compress the liquid, line CD is almost vertical. $ABCD$ is a typical *isotherm* of a pure substance

Table 9.1 *Triple-point data*

Substance	*Temperature, K*	*Pressure, mm Hg*
Acetylene, C_2H_2	192.4	962
Ammonia, NH_3	195.42	45.58
Argon, A	83.78	515.7
Carbon dioxide, CO_2	216.55	3885.1
Carbon monoxide, CO	68.14	115.14
Methane, C_2H_6	89.88	0.006
Ethylene, C_2H_4	104.00	0.9
Hydrogen, H_2	13.84	52.8
Methane, CH_4	90.67	87.7
Nitrogen, N_2	63.15	94.01
Oxygen, O_2	54.35	1.14
Water, H_2O	273.16	4.587

on a $p-v$ diagram. Some isotherms are shown in Fig. 9.5. As the temperature increases, the liquid-vapour transition, as represented by BC, decreases, and becomes zero at the critical point. Below the critical point only, there is a liquid-vapour transition zone, where a saturated liquid, on heating, absorbs the latent heat of vaporization, and becomes saturated vapour at a constant pressure and temperature. Similarly, a saturated vapour, on cooling, releases the latent heat of condensation at constant pressure and temperature to become saturated liquid. Above the critical point, however, a liquid, upon heating, suddenly *flashes* into vapour, or a vapour, upon cooling, suddenly condenses into liquid. There is no distinct transition zone from liquid to vapour and vice versa. The isotherm passing through the critical point is called the *critical isotherm*, and the corresponding temperature is known as the *critical temperature* (t_c). The pressure and volume at the critical point are known as the *critical pressure* (p_c) and the *critical volume* (v_c) respectively. For water

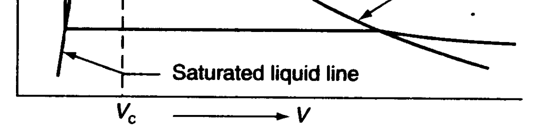


Fig. 9.5 *Saturation curve on p–v diagram*

$$p_c = 221.2 \text{ bar}$$
$$t_c = 374.15°\text{C}$$
$$v_c = 0.00317 \text{ m}^3/\text{kg}$$

The critical point data of certain substances are given in Appendix F. Above the critical point, the isotherms are continuous curves that at large volumes and low pressures approach equilateral hyperbolas.

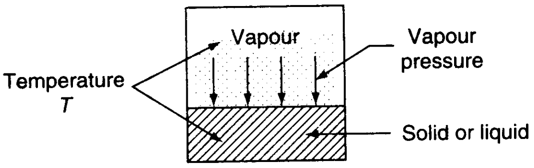


Fig. 9.6 Vapour pressure

When a liquid or solid is in equilibrium with its vapour at a given temperature, the vapour exerts a pressure that depends only on the temperature (Fig. 9.6). In general, the greater the temperature, the higher is the *vapour pressure*. The temperature at which the vapour pressure is equal to 760 mm Hg is called the *normal boiling point*.

Phase change occurs at constant pressure and temperature. A pure liquid at a given pressure will transform into vapour only at a particular temperature, known as *saturation temperature*, which is a function of pressure. Similarly, if the temperature is fixed, the liquid will boil (or condense) only at a particular pressure, called the *saturation pressure*, which is a function of temperature. In Fig. 9.7, if p_1 is the pressure, the corresponding saturation temperature is $(t_{sat})_1$, or if t_2 is the given temperature, the saturation pressure is $(p_{sat})_2$. As the pressure increases, the saturation temperature increases. Saturation states exist up to the critical point. At point A, the liquid starts boiling, and at point B, the boiling gets completed. At A, it is all liquid (saturated) and there is no vapour, while at B, it is all vapour (saturated) and there is no liquid. Vapour content progressively increases as the liquid changes its state from A towards B.

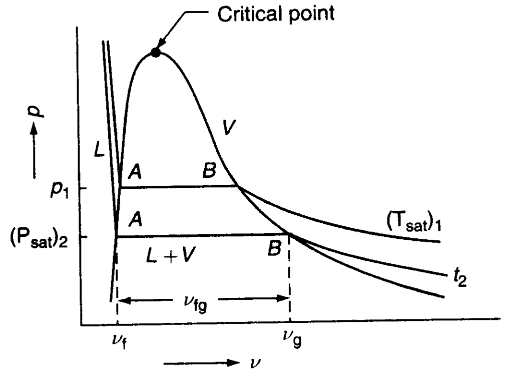


Fig. 9.7 Saturation pressure and temperature

If v_f is the specific volume of the saturated liquid at a given pressure, and v_g the specific volume of the saturated vapour, then $(v_g - v_f)$ or v_{fg} is the change in specific volume during phase transition (boiling or condensation) at the pressure. As pressure increases, v_{fg} decreases, and at the critical point v_{fg} becomes zero.

9.2 p–T DIAGRAM FOR A PURE SUBSTANCE

The state changes of a pure substance, upon slow heating at different constant pressures, are shown on the p–v plane, in Figs 9.2, 9.3, and 9.4. If these state changes are plotted on p–T coordinates, the diagram, as shown in Fig. 9.8, will be obtained. If the heating of ice at −10°C to steam at 250°C at the constant pressure of 1 atm is considered, 1–2 is the solid (ice) heating, 2–3 is the melting of ice at 0°C, 3–4 is the liquid heating, 4–5 is the vaporization of water at 100°C, and 5–6 is the heating in the vapour phase. The process will be reversed from state 6 to state 1 upon cooling. The curve passing through the 2, 3 points is called the *fusion curve*, and the curve passing through the 4, 5 points (which indicate the vaporization or condensation at different temperatures and pressures) is called the *vaporization curve*. If the vapour pressure of a solid is measured at different temperatures, and these are plotted, the *sublimation curve* will be obtained. The fusion curve, the vaporization curve, and the sublimation curve meet at the triple point.

The slopes of the sublimation and vaporization curves for all substances are positive. The slope of the fusion curve for most substances is positive, but for water, it is negative. The temperature at which a liquid boils is very sensitive to pressure, as indicated by the vaporization curve which gives the saturation temperatures at different pressures, but the temperature at which a solid melts is not such a strong function of pressure, as indicated by the small slope of the fusion curve.

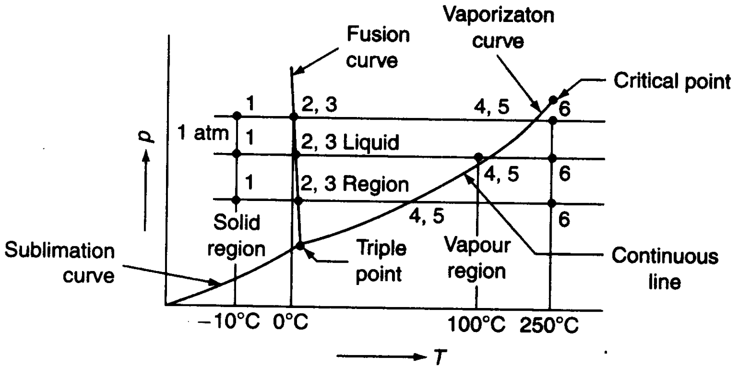
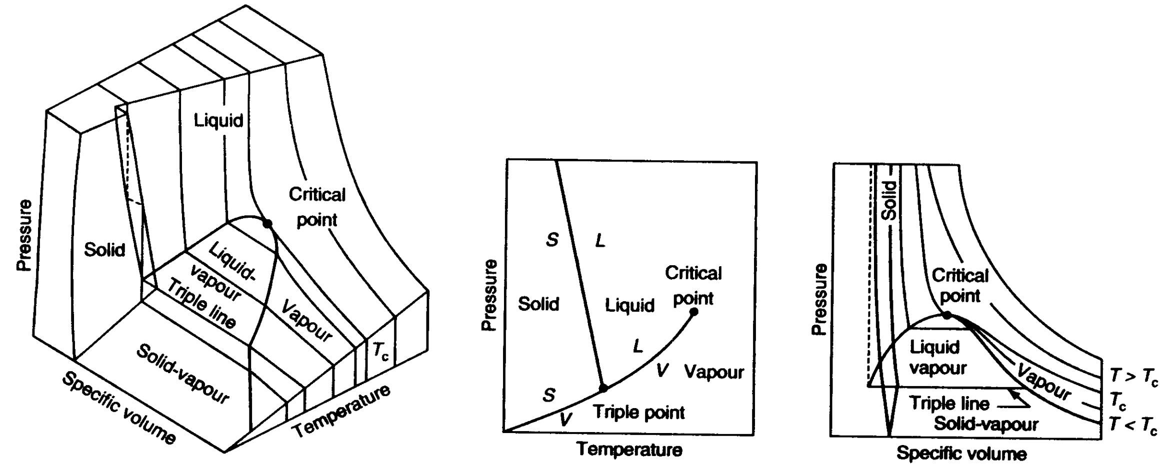


Fig. 9.8 *Phase equilibrium diagram on p–T coordinates*

The triple point of water is at 4.58 mm Hg and 273.16 K, whereas that of CO_2 is at 3885 mm Hg (about 5 atm) and 216.55 K. So when solid CO_2 ('dry ice') is exposed to 1 atm pressure, it gets transformed into vapour directly, absorbing the latent heat of sublimation from the surroundings, which gets cooled or 'refrigerated'.

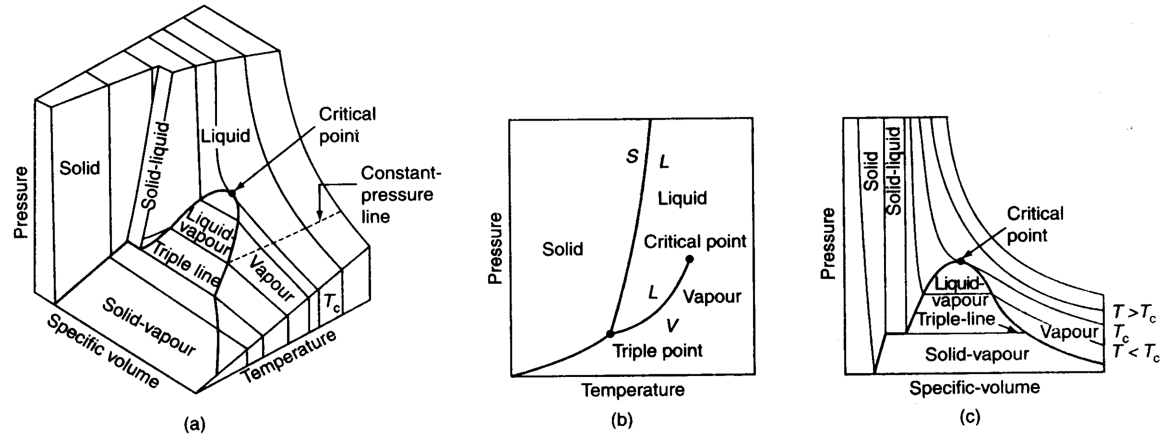
9.3 p–v–T SURFACE

The relationships between pressure, specific volume, and temperature can be clearly understood with the aid of a three-dimensional $p-v-T$ surface. Figure 9.9 illustrates a substance like water that expands upon freezing and Fig. 9.10 illustrates substances other than water which contract upon freezing. The projections on the $p-T$ and $p-v$ planes are also shown in these figures. Any point on the $p-v-T$ surface represents an equilibrium state of the substance. The triple point line when projected to the $p-T$ plane becomes a point. The critical isotherm has a point of inflection at the critical point.



(a) (b) (c)

Fig. 9.9 *p–v–T surface and projections for a substance that expands on freezing (a) Three-dimensional view (b) Phase diagram (c) p–v diagram*



(a) (b) (c)

Fig. 9.10 *p–v–T surface and projections for a substance that contracts on freezing (a) Three-dimensional view (b) Phase diagram (c) p–v diagram*

9.4 T–s DIAGRAM FOR A PURE SUBSTANCE

The heating of the system of 1 kg of ice at −5°C to steam at 250°C is again considered, the pressure being maintained constant at 1 atm. The entropy increases of the system in different regimes of heating are given below.

1. The entropy increase of ice as it is heated from −5°C to 0°C at 1 atm. ($c_{\text{pice}} = 2.093$ kJ/kg K).

$$\Delta s_1 = s_2 - s_1 = \int \frac{\text{đ}Q}{T} = \int_{T_1 = 268}^{T_2 = 273} \frac{mc_p \mathrm{d}T}{T} = c_p \ln \frac{273}{268} = 2.093 \ln \frac{273}{268} = 0.0398 \text{ kJ/kg K.}$$

2. The entropy increase of ice as it melts into water at 0°C (latent heat of fusion of ice = 334.96 kJ/kg)

$$\Delta s_2 = s_3 - s_2 = \frac{334.96}{273} = 1.23 \text{ kJ/kg K}$$

3. The entropy increase of water as it is heated from 0°C to 100°C ($c_{\text{p water}} = 4.187$ kJ/kg K)

$$\Delta s_3 = s_4 - s_3 = c_p \ln \frac{T_3}{T_2} = 4.187 \ln \frac{373}{273} = 1.305 \text{ kJ/kg K}$$

4. The entropy increase of water as it is vaporized at 100°C, absorbing the latent heat of vaporization (2257 kJ/kg)

$$\Delta s_4 = s_5 - s_4 = \frac{2257}{273} = 6.05 \text{ kJ/kg K}$$

5. The entropy increase of vapour as it is heated from 100°C to 250°C at 1 atm

$$\Delta s_5 = s_6 - s_5 = \int_{373}^{523} c_p \frac{\mathrm{d}T}{T} = 2.093 \ln \frac{523}{373} = 0.706 \text{ kJ/kg K}$$

assuming the average specific heat of steam in the temperature range of 100°C to 250°C as 2.093 kJ/kg K.

These entropy changes are shown in Fig. 9.11. The curve 1-2-3-4-5-6 is the isobar of 1 atm. If, during the heating process, the pressure had been maintained constant at 2 atm, a similar curve would be obtained. The states 2, 3, 4, and 5 are saturation states. If these states for different pressures are joined, as in Figs 9.3 and 9.4, the phase equilibrium diagram of a pure substance on the T–s coordinates, as shown in Fig. 9.12, would be obtained.

Most often, liquid-vapour transformations only are of interest, and Fig. 9.13 shows the liquid, the vapour, and the transition zones only. At a particular pressure, s_f is the specific entropy of saturated water, and s_g is that of saturated vapour. The entropy change of the system during the phase change from liquid to vapour at that pressure is s_{fg} $(= s_g - s_f)$. The value of s_{fg} decreases as the pressure increases, and becomes zero at the critical point.

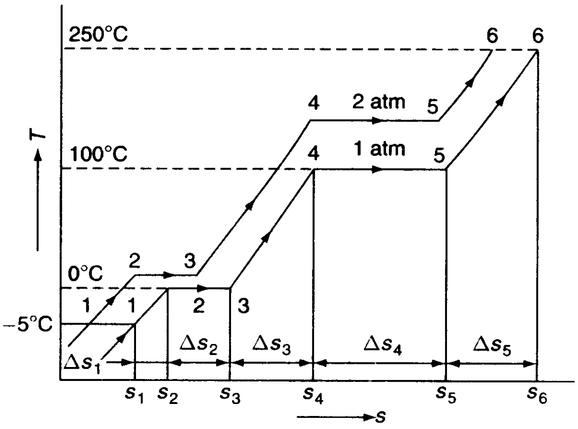


Fig. 9.11 *Isobars on T–s plot*

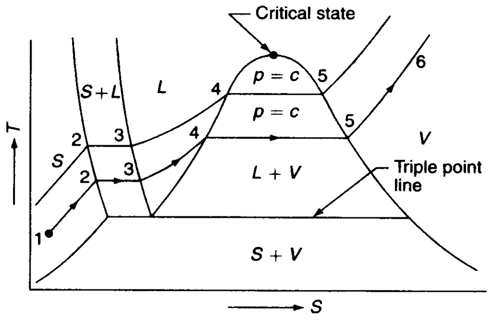


Fig. 9.12 *Phase equilibrium diagram on T–s coordinates*

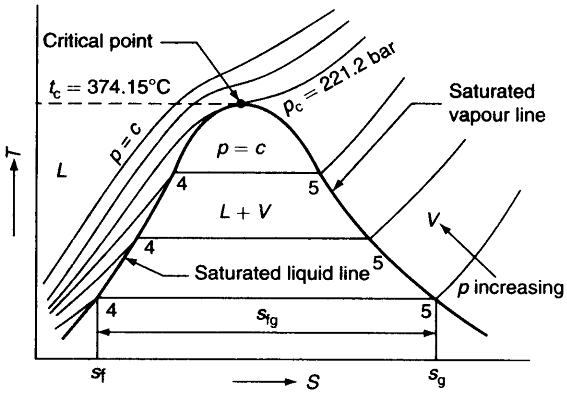


Fig. 9.13 *Saturation (or vapour) dome for water*

9.5 h–s DIAGRAM OR MOLLIER DIAGRAM FOR A PURE SUBSTANCE

From the first and second laws of thermodynamics, the following property relation was obtained.

$$Tds = dh - vdp$$

or

$$\left(\frac{\partial h}{\partial s}\right)_p = T \tag{9.1}$$

This equation forms the basis of the h–s diagram of a pure substance, also called the Mollier diagram. The slope of an isobar on the h–s coordinates is equal to the absolute saturation temperature $(t_{sat} + 273)$ at that pressure. If the temperature remains constant the slope will remain constant. If the temperature increases, the slope of the isobar will increase.

Consider the heating of a system of ice at −5°C to steam at 250°C, the pressure being maintained constant at 1 atm. The slope of the isobar of 1 atm on the h–s coordinates (Fig. 9.14) first increases as the temperature of the ice increases from −5°C to 0°C (1 -2). Its slope then remains constant as ice melts into water at the constant temperature of 0°C (2–3). The slope of the isobar again increases as the temperature of water rises from 0°C to 100°C (3–4). The slope again remains constant as water vaporizes into steam at the constant temperature of 100°C (4–5). Finally, the slope of the isobar continues to increase as the temperature of steam increases to 250°C (5–6) and beyond. Similarly, the isobars of different pressures can be drawn on the h–s diagram as shown in Figs 9.14 and 9.15. States 2, 3, 4 and 5 are saturation states. Figure 9.15 shows the phase equilibrium diagram of a pure substance on the h–s coordinates, indicating the saturated solid line, saturated liquid lines and saturated vapour line, the various phases, and the transition (mixture) zones.

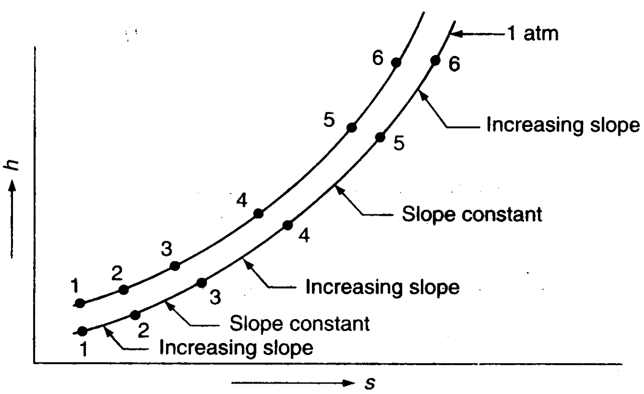


Fig. 9.14 *Isobars on h–s plot*

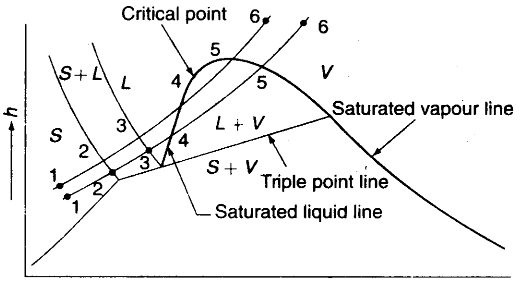


Fig. 9.15 *Phase equilibrium diagram on h–s coordinates (mollier diagram)*

Figure 9.16 is the h–s or the Mollier diagram indicating only the liquid and vapour phases. As the pressure increases, the saturation temperature increases, and so the slope of the isobar also increases. Hence, the *constant pressure lines diverge from one another*, and the critical isobar is a tangent at the critical point, as shown. In the vapour region, the states of equal slopes at various pressures are joined by lines, as shown, which are the *constant temperature lines*. Although the slope of an isobar remains continuous beyond the saturated vapour line, the isotherm bends towards the right and its slope decreases asymptotically to zero, because in the ideal gas region it becomes horizontal and the constant enthalpy implies constant

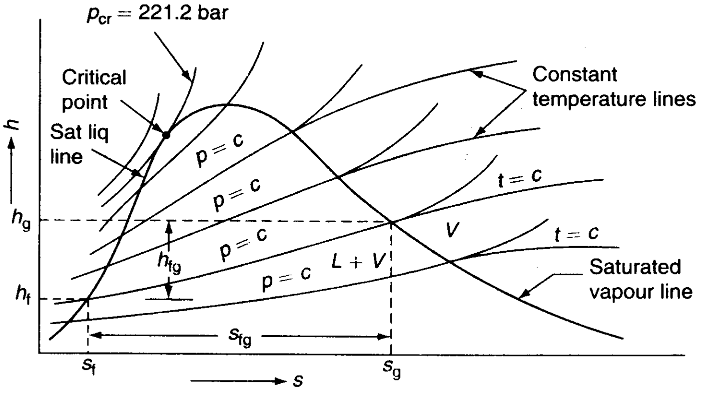


Fig. 9.16 *Enthalpy-entropy diagram of water*

temperature. $\left[\left(\frac{\partial h}{\partial s}\right)_T = T + v\left(\frac{\partial p}{\partial s}\right)_T = T -- v\left(\frac{\partial T}{\partial v}\right)_p\right]$, by Maxwell's relation, Chapter 11. For an ideal gas, $\left(\frac{\partial h}{\partial s}\right)_T = T - v\frac{T}{v} = 0$. For $T = c$, $h = c$]. At a particular pressure, h_f is the specific enthalpy of saturated water, h_g is that of saturated vapour, and h_{fg} (= $h_g - h_f$) is the latent heat of vaporization at that pressure. As the pressure increases, h_{fg} decreases, and at the critical pressure, h_{fg} becomes zero.

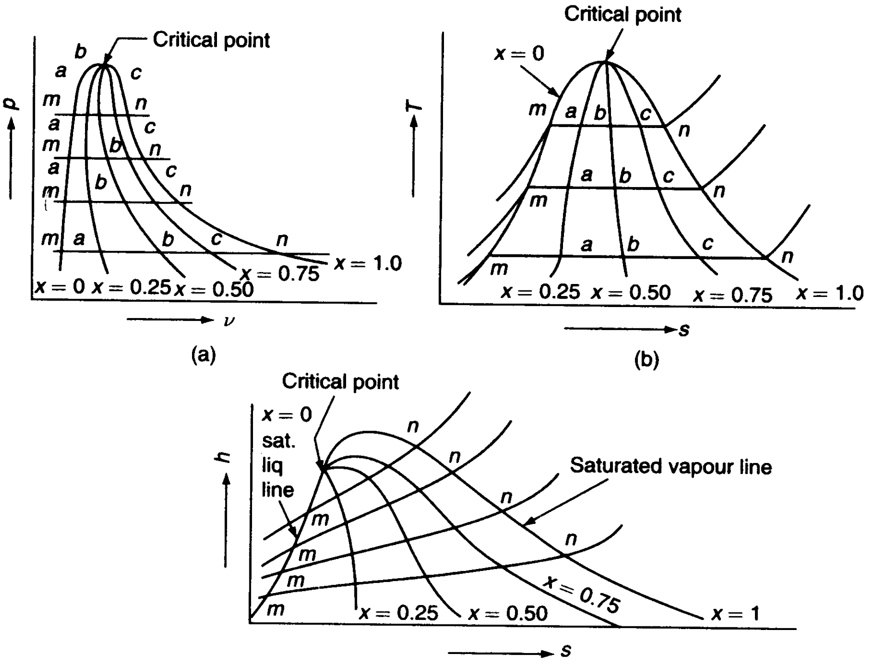
9.6 QUALITY OR DRYNESS FRACTION

If in 1 kg of liquid-vapour mixture, x kg is the mass of vapour and $(1 - x)$ kg is the mass of liquid, then x is known as the *quality* or dryness fraction of the liquid-vapour mixture. Therefore, quality indicates the mass fraction of vapour in a liquid vapour mixture, or

$$x = \frac{m_v}{m_v + m_l}$$

where m_v and m_l are the masses of vapour and liquid respectively in the mixture. The value of x varies between 0 and 1. For saturated water, when water just starts boiling, $x = 0$, and for saturated vapour, when vaporization is complete, $x = 1$, for which the vapour is said to be *dry saturated.*

Points m in Fig. 9.17 (a), (b), and (c) indicate the saturated liquid states with $x = 0$, and points n indicate the saturated vapour states with $x = 1$, the lines mn indicating the transition from liquid to vapour. Points



(a) (b)

(c)

Fig. 9.17 *Constant quality lines on p–v, T–s and h–s diagrams* *a*, *b*, and *c* at various pressures indicate the situations when the masses of vapour reached 25%, 50%, and 75% of the total mass, i.e. at points *a*, the mass of liquid is 75% and the mass of vapour is 25% of the total mass, at points *b*, the mixture consists of 50% liquid and 50% vapour by mass, and at points *c*, the mixture consists of 75% vapour and 25% liquid by mass. The lines passing through points *a*, *b* and *c* are the constant quality lines of 0.25, 0.50, and 0.75 respectively. Constant quality lines start from the critical point.

Let V be the total volume of a liquid vapour mixture of quality x, V_f the volume of the saturated liquid, and V_g the volume of the saturated vapour, the corresponding masses being m, m_f, and m_g respectively.

Now
$$m = m_f + m_g$$

and
$$V = V_f + V_g$$

$$mv = m_f v_f + m_g v_g = (m - m_g) v_f + m_g v_g$$

$$\therefore \quad v = \left(1 - \frac{m_g}{m}\right) v_f + \frac{m_g}{m} v_g$$

$$v = (1 - x) v_f + x v_g \tag{9.2}$$

where $x = \frac{m_g}{m}$, v_f = specific volume of saturated liquid, v_g = specific volume of saturated vapour, and v = specific volume of the mixture of quality x.

Similarly

$$s = (1 - x) s_f + x s_g \tag{9.3}$$

$$h = (1 - x) h_f + x h_g \tag{9.4}$$

$$u = (1 - x) u_f + x u_g \tag{9.5}$$

where s, h, and u refer to the mixture of quality x, the suffix f and suffix g indicate the conditions of saturated liquid and saturated vapour respectively.

From Eq. (9.2)

$$v = (1 - x) v_f + x v_g$$
$$= v_f + x(v_g - v_f)$$
$$= v_f + x\, v_{fg}$$

or
$$v = v_f + x v_{fg} \tag{9.6}$$

Similarly

$$h = h_f + x h_{fg} \tag{9.7}$$

$$s = s_f + x s_{fg} \tag{9.8}$$

$$u = u_f + x u_{fg} \tag{9.9}$$

However,
$$\rho \neq \rho_f + x \rho_{fg}$$

Volume fraction of vapour or voidage, $\alpha = v_g/v$

$$m = m_f + m_g$$

$$\rho v = \rho_f v_f + \rho_g v_g = \rho_f (v - v_g) + \rho_g v_g$$

$$\rho = (1 - \alpha) \rho_f + \alpha \rho_g \tag{9.9a}$$

9.7 STEAM TABLES

The properties of water are arranged in the steam tables as functions of pressure and temperature. Separate tables are provided to give the properties of water in the saturation states and in the liquid and vapour phases. The *internal energy of saturated water at the triple point* ($t = 0.01°\text{C}$) *is arbitrarily chosen to be zero.* Since $h = u + pv$, the enthalpy of saturated water at 0.01°C is slightly positive because of the small value of (pv) term. *The entropy of saturated water is also chosen to be zero at the triple point.*

9.7.1 Saturation States

When a liquid and its vapour are in equilibrium at a certain pressure and temperature, only the pressure or the temperature is sufficient to identify the saturation state. If the pressure is given, the temperature of the mixture gets fixed, which is known as the saturation temperature, or if the temperature is given, the saturation pressure gets fixed. Saturated liquid or the saturated vapour has only one independent variable, i.e. only one property is required to be known to fix up the state. Tables A.1 (a) and A.1 (b) in the appendix give the properties of saturated liquid and saturated vapour. In Table A.1 (a), the independent variable is temperature. At a particular temperature, the values of saturation pressure p, and v_f, v_g, h_f, h_{fg}, h_g, s_f and s_g are given, where v_f, h_f, and s_f refer to the saturated liquid states; v_g, h_g and s_g refer to the saturated vapour state; and v_{fg}, h_{fg}, and s_{fg} refer to the changes in the property values during evaporation (or condensation) at that temperature, where $v_{fg} = v_g - v_f$ and $s_{fg} = s_g - s_f$.

In Table A.1 (b), the independent variable is pressure. At a particular pressure, the values of saturation temperature t, and v_f, v_g, h_f, h_{fg}, h_g, s_f, and s_g are given. Depending upon whether the pressure or the temperature is given, either Table A.1 (a) or Table A.1 (b) can be conveniently used for computing the properties of saturation states.

If data are required for intermediate temperatures or pressures, *linear interpolation is normally accurate.* The reason for the two tables is to reduce the amount of interpolation required.

9.7.2 Liquid-vapour Mixtures

Let us consider a mixture of saturated liquid water and water vapour in equilibrium at pressure p and temperature t. The composition of the mixture by mass will be given by its quality x, and its state will be within the vapour dome (Fig. 9.18). The properties of the mixture are as given in Article 9.6, i.e.

$$v = v_f + xv_{fg}$$

$$u = u_f + xu_{fg}$$

$$h = h_f + xh_{fg}$$

$$s = s_f + xs_{fg}$$

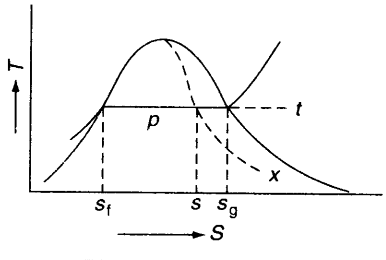


Fig. 9.18 *Property in two phase region*

where v_f, v_{fg}, u_f, u_{fg}, h_f, h_{fg}, s_f and s_{fg} are the saturation properties at the given pressure and temperature.

If p or t and the quality of the mixture are given, the properties of the mixture (v, u, h and s) can be evaluated from the above equations. Sometimes, instead of quality, one of the above properties, say, specific volume v, and pressure or temperature are given. In that case, the quality of the mixture x has to be calculated from the given v and p or t and then x being known, other properties are evaluated.

9.7.3 Superheated Vapour

When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure, the vapour is said to be *superheated* (state 1 in Fig. 9.19). The difference between the temperature of the superheated vapour and the saturation temperature at that pressure is called the *superheat* or the *degree of superheat*. As shown in Fig. 9.19, the difference ($t_1 - t_{sat}$) is the superheat.

In a superheated vapour at a given pressure, the temperature may have different values greater than the saturation temperature. Table A.2 in the appendix gives the values of the properties (volume, enthaly, and entropy) of superheated vapour for each tabulated pair of values of pressure and temperature, both of which are now independent. Interpolation or extrapolation is to be used for pairs of values of pressure and temperature not given.

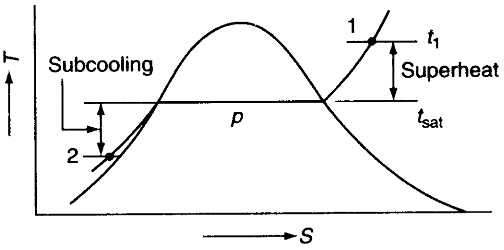


Fig. 9.19 *Superheat and subcooling*

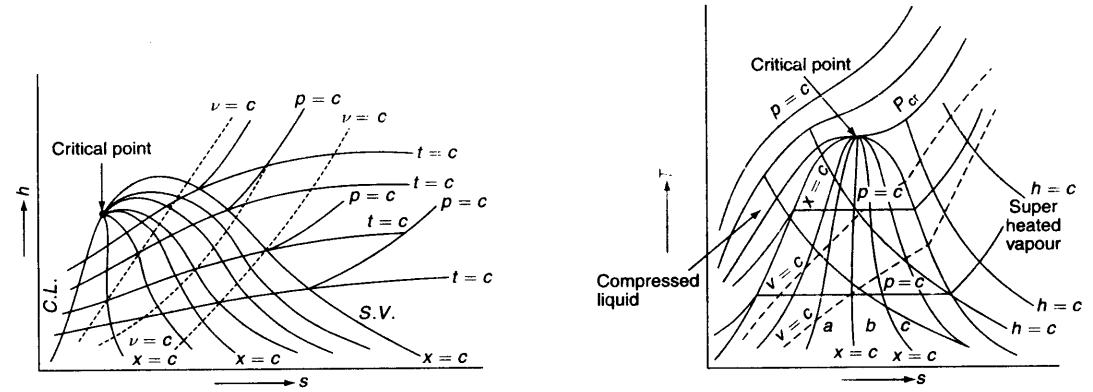
9.7.4 Compressed Liquid

When the temperature of a liquid is less than the saturation temperature at the given pressure, the liquid is called *compressed liquid* (state 2 in Fig. 9.19). The pressure and temperature of compressed liquid may vary independently, and a table of properties like the superheated vapour table could be arranged to give the properties at any *p* and *t*. However, the *properties of liquids vary little with pressure.* Hence the properties are taken from the saturation tables at the temperature of the compressed liquid. When a liquid is cooled below its saturation temperature at a certain pressure it is said to be *subcooled.* The difference in saturation temperature and the actual liquid temperature is known as the degree of subcooling, or simply, subcooling (Fig. 9.17).

9.8 CHARTS OF THERMODYNAMIC PROPERTIES

The presentation of properties of substances in the form of a chart has certain obvious advantages. The manner of variation of properties is clearly demonstrated in the chart and there is no problem of interpolation. However, the precision is not as much as in steam tables.

The temperature-entropy plot and enthalpy-entropy plot (Fig. 9.20a) are commonly used. The temperature-entropy plot shows the vapour dome and the lines of constant pressure, constant volume, constant enthalpy, constant quality, and constant superheat. However, its scale is small and limited in use. The enthalpy-entropy plot or Mollier chart, has a larger scale to provide data suitable for many computations. It contains the same



(b)

(a)

Fig. 9.20 *Constant property lines on (a) T–s plot and (b) Mollier diagram* data as does the T–s chart. The Mollier chart for water is given in Appendix F.I. The Mollier diagram for steam with data taken from Keenan et al. Steam Tables (John Willey, N.Y., 1969 is given in Fig. 9.21.)

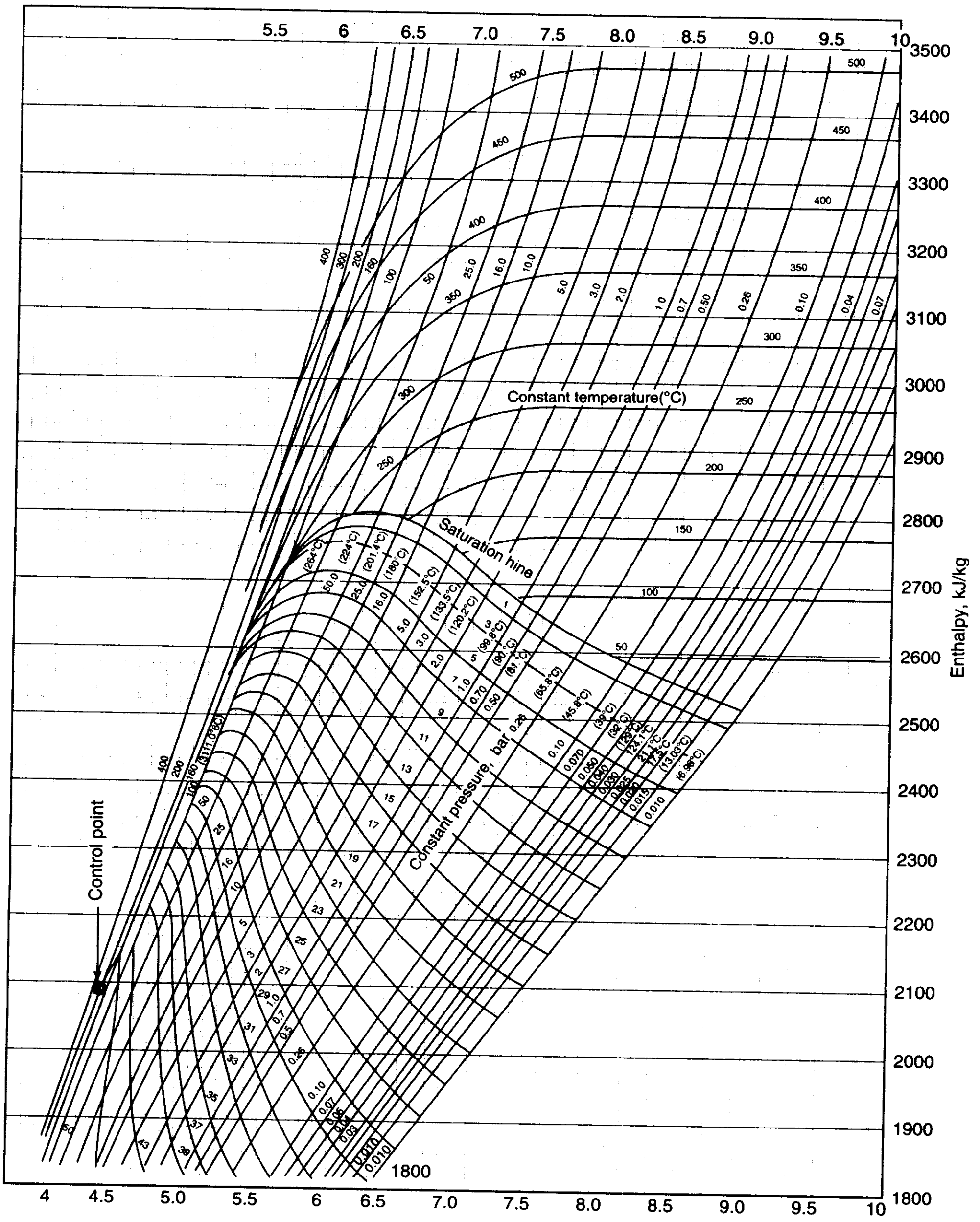


Fig. 9.21 *Mollier diagram for steam (Data taken from Keenan, J.H., F.G. keyes, P.C. Hill and J.g. Moore, Steam Tables, John Wiley, N.Y., 1969)*

9.9 MEASUREMENT OF STEAM QUALITY

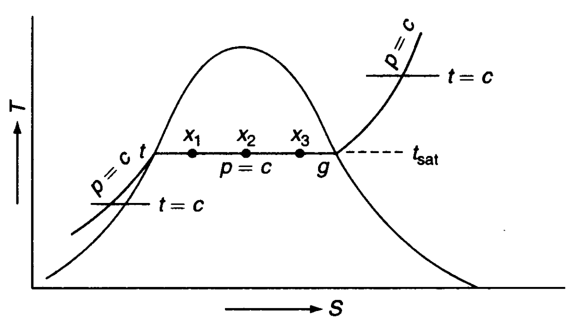
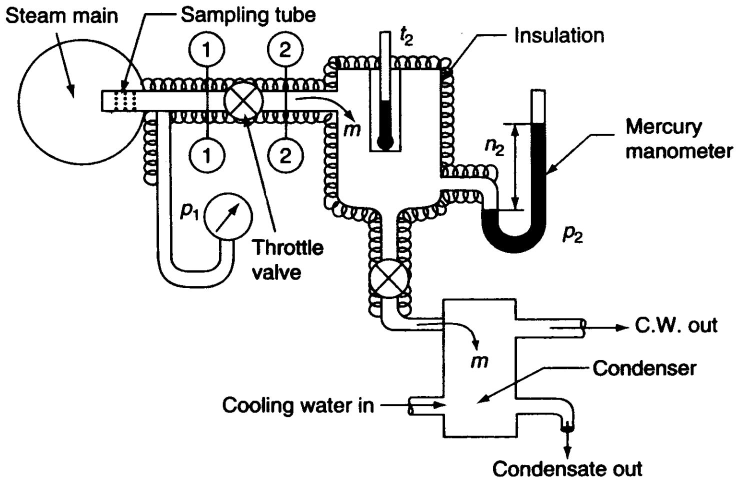


Fig. 9.22 *Quality of liquid-vapour mixture*

The state of a pure substance gets fixed if two independent properties are given. A pure substance is thus said to have two degrees of freedom. Of all thermodynamic properties, it is easiest to measure the pressure and temperature of a substance. Therefore, whenever pressure and temperature are independent properties, it is the practice to measure them to determine that state of the substance. This is done in the compressed liquid region or the superheated vapour region (Fig. 9.22), where the measured values of pressure and temperature would fix up the state. But when the substance is in the saturation state or two-phase region (Fig. 9.22), the measured values of pressure and temperature could apply equally well to saturated liquid point f, saturated vapour point g, or to mixtures of any quality, points x_1, x_2 or x_3. Of the two properties, p and t, only one is independent; the other is a dependent property. If pressure is given, the saturation temperature gets automatically fixed for the substance. In order to fix up the state of the mixture, apart from either pressure or temperature, one more property, such as specific volume, enthalpy or composition of the mixture (quality) is required to be known. Since it is relatively difficult to measure the specific volume of a mixture, devices such as calorimeters are used for determining the quality or the enthalpy of the mixture.

In the measurement of quality, the object is always to bring the state of the substance from the two-phase region to the single-phase or superheated region, where both pressure and temperature are independent, and measured to fix the state, either by adiabatic throttling or electric heating.

In the *throttling calorimeter*, a sample of wet steam of mass m and at pressure p_1 is taken from the steam main through a perforated sampling tube (Fig. 9.23). Then it is throttled by the partially-opened valve (or orifice) to a pressure p_2, measured by mercury manometer, and temperature t_2, so that after throttling



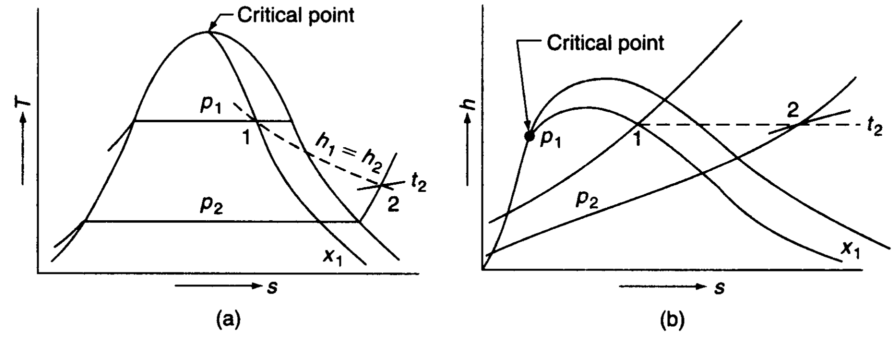
[illegible] *Throttling calorimeter*

the steam is in the superheated region. The process is shown on the T–s and h–s diagrams in Fig. 9.24. The steady flow energy equation gives the enthalpy after throttling as equal to enthalpy before throttling. The initial and final equilibrium states 1 and 2 are joined by a dotted line since throttling is irreversible (adiabatic but not isentropic) and the intermediate states are non-equilibrium states not describable by thermodynamic coordinates. The initial state (wet) is given by p_1 and x_1, and the final state by p_2 and t_2 (superheated). Now

since
$$h_1 = h_2$$
$$h_{\text{fp1}} + x_1 h_{\text{fgp1}} = h_2$$

or
$$x_1 = \frac{h_2 - h_{\text{fp1}}}{h_{\text{fgp1}}}$$

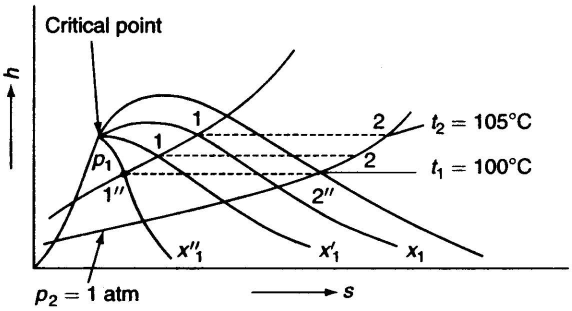
With p_2 and t_2 being known, h_2 can be found out from the superheated steam table. The values of h_f and h_fg are taken from the saturated steam table corresponding to pressure p_1. Therefore, the quality of the wet steam x_1 can be calculated.



(a) (b)

Fig. 9.24 *Throttling process on T–s and h–s plots*

To be sure that steam after throttling is in the single-phase or superheated region, a minimum of 5°C superheat is desired. So if the pressure after throttling is given and the minimum 5°C superheat is prescribed, then there is the minimum quality of steam (or the maximum moisture content) at the given pressure p_1 which can be measured by the throttling calorimeter. For example, if p_2 = 1 atm., then t_2 = 105°C and the state 2 after throttling gets fixed as shown in Fig. 9.25. From state 2, the constant enthalpy line intersects the constant pressure p_1 line at 1. Therefore, the quality x_1 is the minimum quality that can be measured simply by throttling. If the quality is, say, x'_1 less than x_1, then after throttling to p_2 = 1 atm., the superheat after throttling is less than 5°C. If the quality is x''_1, then throttling to 1 atm. does not give any superheat at all.



[illegible] *Minimum quality that can be measured only by throttling*

When the steam is very wet and the pressure after throttling is not low enough to take the steam to the superheated region, then a *combined separating* and *throttling calorimeter* is used for the measurement of quality. Steam from the main is first passed through a separator (Fig. 9.26), where some part of the moisture separates

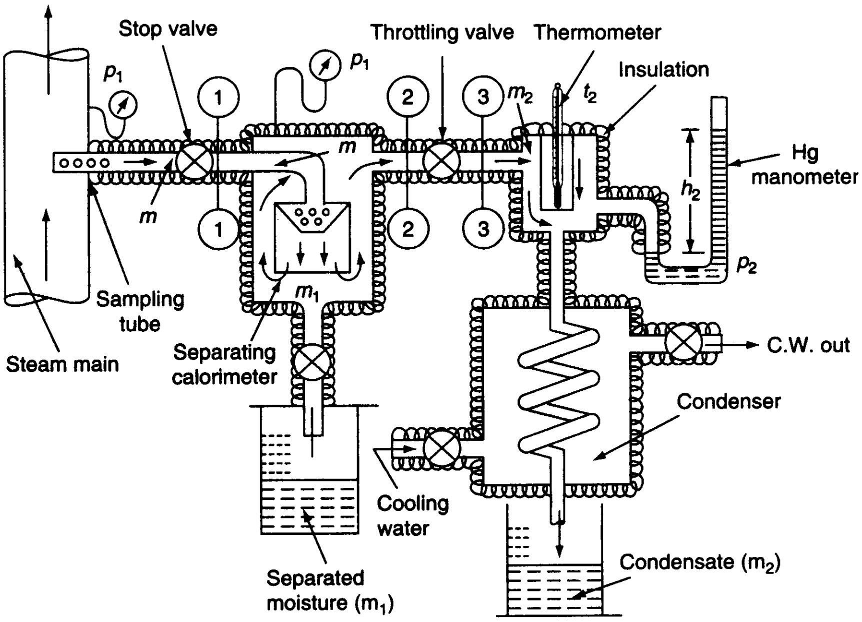


Fig. 9.26 *Separating and throttling calorimeter*

out due to the sudden change in direction and falls by gravity, and the partially dry vapour is then throttled and taken to the superheated region. In Fig. 9.27, process 1–2 represents the moisture separation from the wet sample of steam at constant pressure p_1 and process 2–3 represents throttling to pressure p_2. With p_2 and t_3 being measured, h_3 can be found out from the superheated steam table.

Now,

$$h_3 = h_2 = h_{fp1} + x_2 h_{fgp1}$$

Therefore, x_2, the quality of steam after partial moisture separation, can be evaluated. If m kg of steam is taken through the sampling tube in t secs, m_1 kg of it is separated, and m_2 kg is throttled and then condensed to water and collected, then $m = m_1 + m_2$, and at state 2, the mass of dry vapour will be $x_2\ m_2$. Therefore, the quality of the sample of steam at state 1, x_1 is given by

$$x_1 = \frac{\text{mass of dry vapour at state 1}}{\text{mass of liquid-vapour mixture at state 1}} = \frac{x_2 m_2}{m_1 + m_2}$$

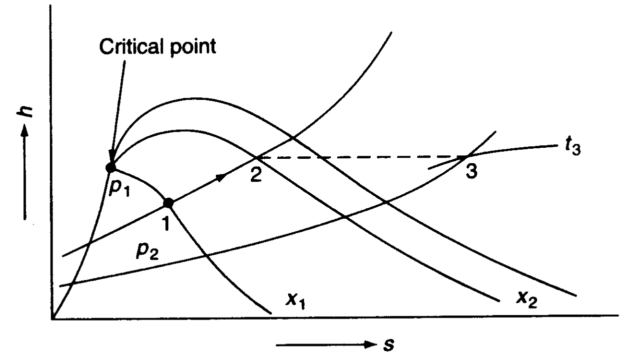


Fig. 9.27 *Separating and throttling processes on h-s plot*

The quality of wet steam can also be measured by an *electric calorimeter* (Fig. 9.28). The sample of steam is passed in steady flow through an electric heater, as shown. The electrical energy input Q should be sufficient to take the steam to the superheated region where pressure p_2 and temperature t_2 are measured. If I is

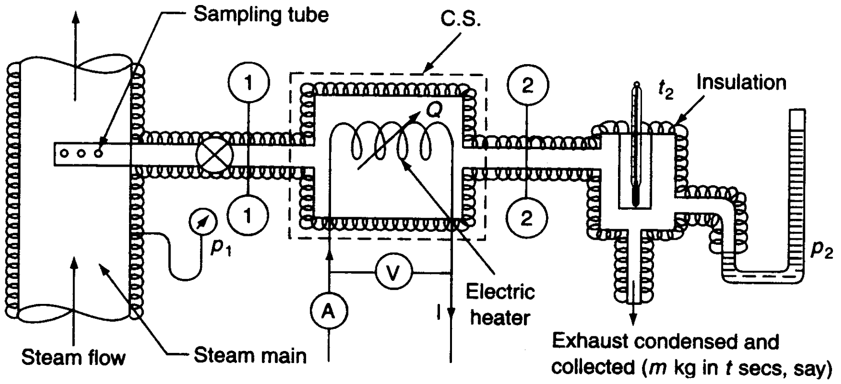


Fig. 9.23 *Electrical calorimeter*

the current flowing through the heater in amperes and V is the voltage across the coil, then at steady state $Q = VI \times 10^{-3}$ kW. If m is the mass of steam taken in t seconds under steady flow condition, then the steady flow energy equation for the heater (as control volume) gives

$$w_1 h_1 + Q = w_1 h_2$$

where w_1 is the steam flow rate in kg/s $\left(w_1 = \frac{m}{t} \text{ kg/s}\right)$

$$\therefore \qquad h_1 + \frac{Q}{w_1} = h_2$$

With h_2, Q and w_1 being known, h_1 can be computed. Now

$$h_1 = h_{\text{fp1}} + x_1 h_{\text{fgp1}}$$

Hence x_1 can be evaluated.

Solved Examples

Example 9.1

Find the saturation temperature, the changes in specific volume and entropy during evaporation, and the latent heat of vaporization of steam at 1 *MPa.*

Solution At 1 MPa, from Table A.1(b) in the Appendix

$$t_{\text{sat}} = 179.91°\text{C}$$

$$v_f = 0.001127 \text{ m}^3/\text{kg} \qquad \textit{Ans.}$$

$$v_g = 0.19444 \text{ m}^3/\text{kg}$$

$$\therefore \quad v_{fg} = v_g - v_f = 0.1933 \text{ m}^3/\text{kg}$$

$$s_f = 2.1387 \text{ kJ/kg K}$$

$$s_g = 6.5865 \text{ kJ/kg K}$$

$$\therefore \quad s_{fg} = s_g - s_f = 4.4478 \text{ kJ/kg K} \qquad \textit{Ans.}$$

$$h_{fg} = h_g - h_f = 2015.3 \text{ kJ/kg} \qquad \textit{Ans.}$$

Example 9.2

Saturated steam has an entropy of 6.76 kJ/kg K. What are its pressure, temperature, specific volume, and enthalpy?

Solution In Table A.1(b), when $s_g = 6.76$ kJ/kg K

$$p = 0.6 \text{ MPa}, \qquad t = 158.85°\text{C}$$

$$v_g = 0.3156 \text{ m}^3/\text{kg}, \quad \text{and} \quad h_g = 2756.8 \text{ kJ/kg} \qquad \textit{Ans.}$$

Example 9.3

Find the enthalpy and entropy of steam when the pressure is 2 MPa and the specific volume is 0.09 m³/kg.

Solution In Table A.1(b), when $p = 2$ MPa, $v_f = 0.001177$ m³/kg and $v_g = 0.09963$ m³/kg. Since the given volume lies between v_f and v_g, the substance will be a mixture of liquid and vapour, and the state will be within the vapour dome. When in the two-phase region, the composition of the mixture or its quality has to be evaluated first. Now

$$v = v_f + x\, v_{fg}$$

$$0.09 = 0.001177 + x\,(0.09963 - 0.001177)$$

or $$x = 0.904 \text{ or } 90.4\%$$

At $$2 \text{ MPa}, h_f = 908.79 \quad \text{and} \quad h_{fg} = 1890.7 \text{ kJ/kg}$$

$$s_f = 2.4474 \quad \text{and} \quad s_{fg} = 3.8935 \text{ kJ/kg K}$$

$$h = h_f + x\, h_{fg} = 908.79 + 0.904 \times 1890.7 = 2618.79 \text{ kJ/kg} \qquad \textit{Ans.}$$

$$s = s_f + x\, s_{fg} = 2.4474 + 0.904 \times 3.8935 = 5.9534 \text{ kJ/kg K} \qquad \textit{Ans.}$$

Example 9.4

Find the enthalpy, entropy, and volume of steam at 1.4 MPa, 380°C.

Solution At $p = 1.4$ MPa, in Table A.1(b), $t_{sat} = 195.07°$C. Therefore, the state of steam must be in the superheated region. In Table A.2, for properties of superheated steam,

at 1.4 MPa, 350°C $\quad v = 0.2003$ m³/kg

$h = 3149.5$ kJ/kg

$s = 7.1360$ kJ/kg K

and at 1.4 MPa, 400°C $\quad v = 0.2178$ m³/kg

$h = 3257.5$ kJ/kg

$s = 7.3026$ kJ/kg K

∴ By interpolation

at 1.4 MPa, 380°C $v = 0.2108$ m³/kg

$h = 3214.3$ kJ/kg

$s = 7.2360$ kJ/kg K *Ans.*

Example 9.5

A vessel of volume 0.04 *m*³ *contains a mixture of saturated water and saturated steam at a temperature of* 250°*C. The mass of the liquid present is* 9 *kg. Find the pressure, the mass, the specific volume, the enthalpy, the entropy, and the internal energy.*

Solution From Table A.1(a), at 250°C $p_{sat} = 3.973$ MPa

$v_f = 0.0012512$ m³/kg, $v_g = 0.05013$ m³/kg

$h_f = 1085.36$ kJ/kg, $h_{fg} = 1716.2$ kJ/kg

$s_f = 2.7927$ kJ/kg K, $s_{fg} = 3.2802$ kJ/kg K

Volume of liquid, $V_f = m_f v_f = 9 \times 0.0012512 = 0.01126$ m³

Volume of vapour, $V_g = 0.04 - 0.01126 = 0.02874$ m³

∴ Mass of vapour

$$m_g = \frac{V_g}{v_g} = \frac{0.02874}{0.05013} = 0.575 \text{ kg}$$

∴ Total mass of mixture,

$$m = m_f + m_g = 9 + 0.575 = 9.575 \text{ kg}$$ *Ans.*

Quality of mixture,

$$x = \frac{m_g}{m_f + m_g} = \frac{0.575}{9.575} = 0.06$$

∴ $v = v_f + xv_{fs} = 0.0012512 + 0.06\,(0.05013 - 0.0012512)$

$= 0.00418$ m³/kg *Ans.*

$h = h_f + xh_{fg} = 1085.36 + 0.06 \times 1716.2 = 1188.32$ kJ/kg *Ans.*

$s = s_f + xs_{fg} = 2.7927 + 0.06 \times 3.2802 = 2.9895$ kJ/kg K *Ans.*

$u = h - pv$

$= 1188.32 - 3.973 \times 10^3 \times 0.00418 = 1171.72$ kJ/kg *Ans.*

Also, at 250°C, $u_f = 1080.39$ and $u_{fg} = 1522.0$ kJ/kg

∴ $u = u_f + xu_{fg} = 1080.39 + 0.06 \times 1522 = 1071.71$ kJ/kg *Ans.*

Example 9.6

Steam initially at 0.3 *MPa,* 250°*C is cooled at constant volume.* (*a*) *At what temperature will the steam become saturated vapour?* (*b*) *What is the quality at* 80°*C? What is the heat transferred per kg of steam in cooling from* 250°*C to* 80°*C?*

Solution At 0.3 MPa, $t_{sat} = 133.55°C$

Since $t > t_{sat}$, the state would be in the superheated region (Fig. Ex. 9.6). From Table A.2, for properties of superheated steam, at 0.3 MPa, 250°C

$$v = 0.7964 \text{ m}^3\text{/kg}$$
$$h = 2967.6 \text{ kJ/kg}$$
$$\therefore \quad v_1 = v_3 = v_2 = 0.7964 \text{ m}^3\text{/kg}$$

In Table A.1

when $\quad v_g = 0.8919, \quad t_{sat} = 120°C$

when $\quad v_g = 0.7706, \quad t_{sat} = 125°C$

Therefore, when $v_g = 0.7964$, t_{sat}, by linear interpolation, would be 123.9°. Steam would become saturated vapour at $t = 123.9°C$ *Ans.* (a)

At 80°C, $v_f = 0.001029$ m³/kg, $v_g = 3.407$ m³/kg, $h_f = 334.91$ kJ/kg, $h_{fg} = 2308.8$ kJ/kg,

$p_{sat} = 47.39$ kPa $v_1 = v_2 = 0.7964 \text{ m}^3\text{/kg} = v_{f80°C} + x_2 v_{fg80°C}$

$$= 0.001029 + x_2(3.407 - 0.001029)$$

$$\therefore \quad x_2 = \frac{0.79539}{3.40597} = 0.234 \qquad \textit{Ans. (b)}$$

$$h_2 = 334.91 + 0.234 \times 2308.8 = 875.9 \text{ kJ/kg}$$
$$h_1 = 2967.6 \text{ kJ/kg}$$

From the first law of thermodynamics

$$đQ = du + pdv$$
$$\therefore \quad (đQ)_v = du$$
$$\text{or} \quad Q_{1-2} = u_2 - u_1 = (h_2 - p_2 v_2) - (h_1 - p_1 v_1)$$
$$= (h_2 - h_1) + v(p_1 - p_2)$$
$$= (875.9 - 2967.6) + 0.7964\,(300 - 47.39)$$
$$= 2091.7 + 201.5$$
$$= -1890.2 \text{ kJ/kg}$$

Ans. (c)

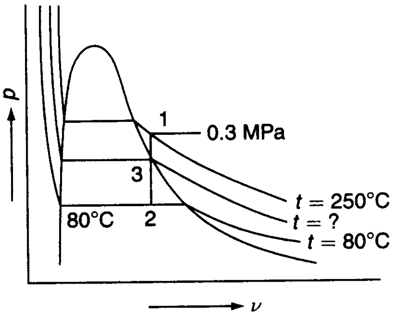


Fig. Ex. 9.6

Example 9.7

Steam initially at 1.5 MPa, 300°C expands reversibly and adiabatically in a steam turbine to 40°C. Determine the ideal work output of the turbine per kg of steam.

Solution The steady flow energy equation for the control volume, as shown in Fig. Ex. 9.7.1, gives (other energy terms being neglected)

$$h_1 = h_2 + W$$
$$\therefore \quad W = h_1 - h_2$$

Work is done by steam at the expense of a fall in its enthalpy value. The process is reversible and adiabatic, so it is isentropic. The process is shown on the T–s and h–s diagrams in Fig. Ex. 9.7.2.

From Table A.1(a), at 40°C

$p_{sat} = 7.384$ kPa, $\quad s_f = 0.5725$, and $s_{fg} = 7.6845$ kJ/kg K

$h_f = 167.57$, and $h_{fg} = 2406.7$ kJ/kg

At $p = 1.5$ MPa, $t = 300°C$, from the tabulated properties of superheated steam (Table A.2)

$$s_1 = 6.9189 \text{ kJ/kg K}$$

$$h_1 = 3037.6 \text{ kJ/kg}$$

Since $s_1 = s_2$

$$6.9189 = s_f + x_2\, s_{fg\,40°C} = 0.5725 + x_2 \times 7.6845$$

$$x_2 = \frac{6.3464}{7.6845} = 0.826 \text{ or } 82.6\%$$

$$\therefore \quad h_2 = h_{f40°C} + x_2\, h_{fg\,40°C} = 167.57 + 0.826 \times 2406.7 = 2152.57 \text{ kJ/kg}$$

$$\therefore \quad W = h_1 - h_2 = 3037.6 - 2152.57 = 885.03 \text{ kJ/kg}$$

Ans.

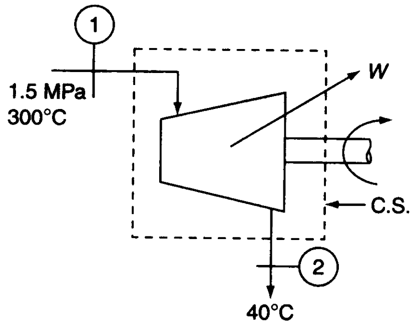
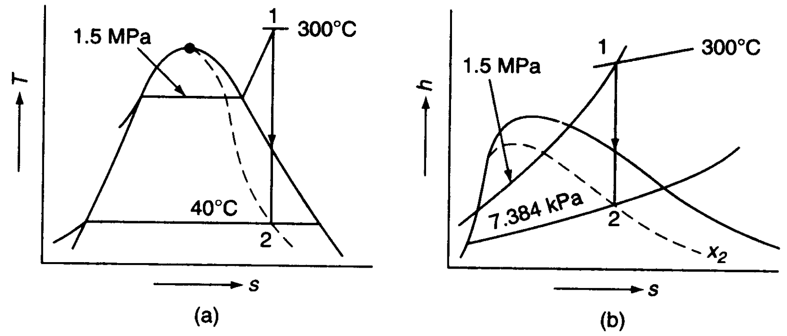


Fig. Ex. 9.7.1



(a) (b)

Fig. Ex. 9.7.2

Example 9.8

Steam at 0.8 MPa, 250°C and flowing at the rate of 1 kg/s passes into a pipe carrying wet steam at 0.8 MPa, 0.95 dry. After adiabatic mixing the flow rate is 2.3 kg/s. Determine the condition of steam after mixing.

The mixture is now expanded in a frictionless nozzle isentropically to a pressure of 0.4 MPa. Determine the velocity of the steam leaving the nozzle. Neglect the velocity of steam in the pipeline.

Solution Figure Ex. 1 gives the flow diagram.

$$w_2 = w_3 - w_1 = 2.3 - 1.0 = 1.3 \text{ kg/s}$$

The energy equation for the adiabatic mixing of the two streams gives

$$w_1 h_1 + w_2 h_2 = w_3 h_3 \tag{1}$$

At 0.8 MPa, 250°C, $h_1 = 2950.0$ kJ/kg

At 0.8 MPa, 0.95 dry

$$h_2 = h_f + 0.95\, h_{fg} = 721.11 + 0.95 \times 2048.0 = 2666.71 \text{ kJ/kg}$$

$\therefore$ From Eq. (1)

$$1 \times 2950 + 1.3 \times 2666.71 = 2.3 \times h_3$$

$$\therefore \quad h_3 = 2790 \text{ kJ/kg}$$

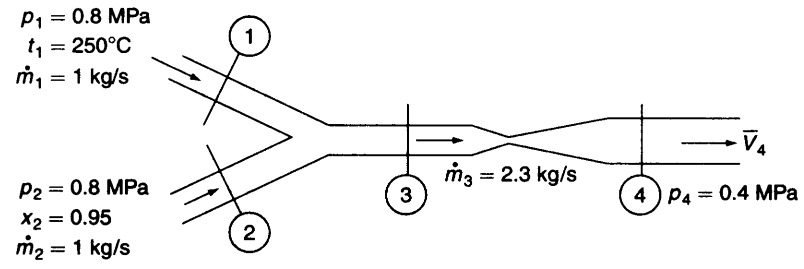


Fig. Ex. 9.11

Since $\qquad (h_g)_{0.8\text{ MPa}} = 2769.1$ kJ/kg

and $h_3 > h_g$, the state must be in the superheated region. From the steam tables, when $p = 0.8$ MPa, $t = 200°C$

$$h = 2839.3 \text{ kJ/kg}$$

When $\qquad p = 0.8 \text{ MPa}, \quad t_{sat} = 170.43°C$

$$h_g = 2769.1 \text{ kJ/kg}$$

By linear interpolation

$$t_3 = 179°C$$

∴ Degree of superheat = 179 – 170.33 = 8.57°C

∴ Condition of steam after mixing = 0.8 MPa, 179°C *Ans.*

The energy equation for the nozzle gives

$$h_3 = h_4 + \frac{V_4^2}{2}$$

since $V_3 = -$ velocity of steam in the pipeline = 0

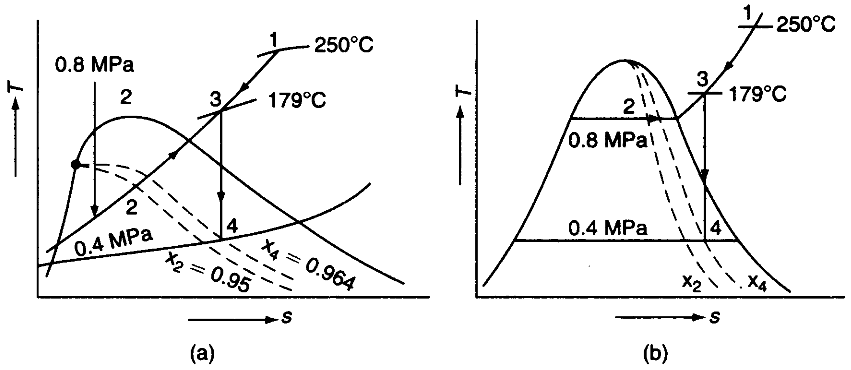
Steam expands isentropically in the nozzle to 0.4 MPa. By interpolation,

$$s_3 = 6.7087 \text{ kJ/kg K} = s_4$$

$$\therefore \quad 6.7087 = 1.7766 + x_4 \times 5.1193$$

$$x_4 = 0.964$$

$$\therefore \quad h_4 = 604.74 + 0.964 \times 2133.8 = 2660 \text{ kJ/kg}$$



(a) (b)

Fig. Ex. 9.12

$$V_4^2 \times 10^{-3} = 2(h_3 - h_4) = 2 \times 130 = 260$$

$$V_4 = \sqrt{26} \times 100 = 509.9 \text{ m/s} \qquad \textit{Ans.}$$

The processes are shown on the h–s and T–s diagrams in Fig. Ex. 9.8.2.

Example 9.9

Steam flows in a pipeline at 1.5 *MPa. After expanding to* 0.1 *MPa in a throttling calorimeter, the temperature is found to be* 120°C. *Find the quality of steam in the pipeline. What is the maximum moisture at* 1.5 *MPa that can be determined with this set-up if at least* 5°C *of superheat is required after throttling for accurate readings?*

Solution At state 2 (Fig. Ex. 9.9), when $p = 0.1$ MPa, $t = 120°\text{C}$ by interpolation

$$h_2 = 2716.2 \text{ kJ/kg, and at } p = 1.5 \text{ MPa}$$

$$h_f = 844.89 \quad \text{and} \quad h_{fg} = 1947.3 \text{ kJ/kg}$$

Now, $h_1 = h_2$

or $h_{f1.5\text{MPa}} + x_1 h_{fg1.5\text{MPa}} = h_2$

$$844.89 + x_1 \times 1947.3 = 2716.2$$

$$x_1 = \frac{1871.3}{1947.3} = 0.963 \qquad \textit{Ans.}$$

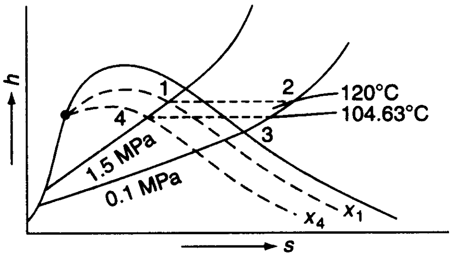
When $p = 0.1$ MPa and $t = 99.63 + 5 = 104.63°\text{C}$

$$h_3 = 2685.5 \text{ kJ/kg}$$

Since $h_3 = h_4$

$$2685.5 = 844.89 + x_4 \times 1947.3$$

$$\therefore \quad x_4 = \frac{1840.6}{1947.3} = 0.948$$



[redacted]

The maximum moisture that can be determined with this set-up is only 5.2%. *Ans.*

Example 9.10

The following data were obtained with a separating and throttling calorimeter:

Pressure in pipeline	1.5 *MPa*
Condition after throttling	0.1 *MPa,* 110°C
During 5 *min moisture collected in the separator*	0.150 *litre at* 70°C
Steam condensed after throttling during 5 *min*	3.24 *kg*

Find the quality of steam in the pipeline

Solution As shown in Fig. Ex. 9.10

at 0.1 MPa, 110°C $\quad h_3 = 2696.2$ kJ/kg

Now $\quad h_3 = h_2 = h_{f1.5\text{MPa}} + x_2\, h_{fg1/5\text{MPa}}$

or $\quad 2696.2 = 844.89 + x_2 \times 1947.3$

$$\therefore \quad x_2 = \frac{1851.31}{1947.3} = 0.955$$

If m_1 = mass of moisture collected in separator in 5 min and m_2 = mass of steam condensed after throttling in 5 min.

then $$x_1 = \frac{x_2 m_2}{m_1 + m_2}$$

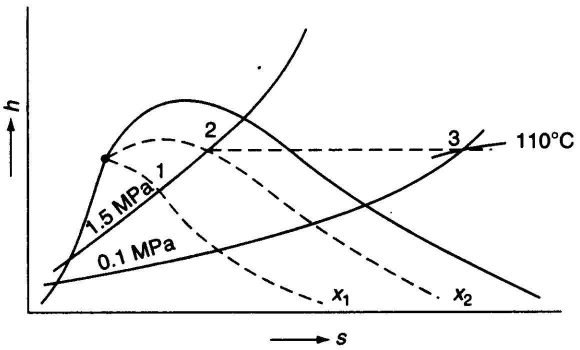
At 70°C, $v_f = 0.001023\ \text{m}^3/\text{kg}$

$$m_1 = \frac{150 \times 10^{-6}\ \text{m}^3}{1023 \times 10^{-6}\ \text{m}^3/\text{kg}} = 0.1462\ \text{kg}$$

$$m_2 = 3.24\ \text{kg}$$

$$\therefore \quad x_1 = \frac{0.955 \times 3.24}{0.1462 + 3.24} = \frac{3.1}{3.3862} = 0.915$$

Ans.



[illegible]

Example 9.11

A steam boiler initially contains 5 m³ of steam and 5 m³ of water at 1 MPa. Steam is taken out at constant pressure until 4 m³ of water is left. What is the heat transferred during the process?

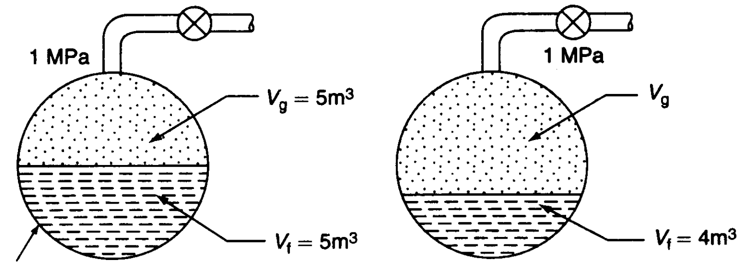
Solution At 1 MPa,

$$v_f = 0.001127, \quad \text{and} \quad v_g = 0.1944\ \text{m}^3/\text{kg}$$
$$h_g = 2778.1\ \text{kJ/kg}$$
$$u_f = 761.68, \quad u_g = 2583.6\ \text{kJ/kg}$$
$$u_{fg} = 1822\ \text{kJ/kg}$$

The initial mass of saturated water and steam in the boiler (Fig. Ex. 9.11).

$$\frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{5}{0.001127} + \frac{5}{0.1944} = (4.45 \times 10^3 + 25.70)\ \text{kg}$$

where suffix *f* refers to saturated water and suffix *g* refers to saturated vapour. Final mass of saturated water and steam



Initial (a)

Final (b)

[illegible]

$$= \frac{4}{0.001127} + \frac{6}{0.1944} = (3.55 \times 10^3 + 30.80)\ \text{kg}$$

$\therefore$ Mass of steam taken out of the boiler (m_s)

$$= (4.45 \times 10^3 + 25.70) - (3.55 \times 10^3 + 30.80)$$
$$= 0.90 \times 10^3 - 5.1 = 894.9\ \text{kg}$$

Making an energy balance, we have: Initial energy stored in saturated water and steam + Heat transferred from the external source = Final energy stored in saturated water and steam + Energy leaving with the steam.

or $$U_1 + Q = U_f + m_s h_g$$

assuming that the steam taken out is dry ($x = 1$)

or $$4.45 \times 10^3 \times 761.68 + 25.70 \times 2583.6 + Q = 3.55 \times 10^3 \times 761.68 + 30.8 \times 2583.6 + 894.9 \times 2778.1$$

or $$Q = 894.9 \times 2778.1 - (0.90 \times 10^3) \times 761.68 + 5.1 \times 2583.6$$
$$= 2425000 - 685500 + 13176$$
$$= 1752{,}676\ \text{kJ} = 1752.676\ \text{MJ}$$ *Ans.*

Example 9.12

A 280 *mm diameter cylinder fitted with a frictionless leakproof piston contains* 0.02 *kg of steam at a pressure of* 0.6 *MPa and a temperature of* 200°*C. As the piston moves slowly outwards through a distance of* 305 *mm, the steam undergoes a fully-resisted expansion during which the steam pressure p and the steam volume V are related by* pV^n = *constant, where n is a constant. The final pressure of the steam is* 0.12 *MPa. Determine* (*a*) *the value of n,* (*b*) *the work done by the steam, and* (*c*) *the magnitude and sign of heat transfer.*

Solution Since the path of expansion (Fig. Ex. 9.12) follows the equation

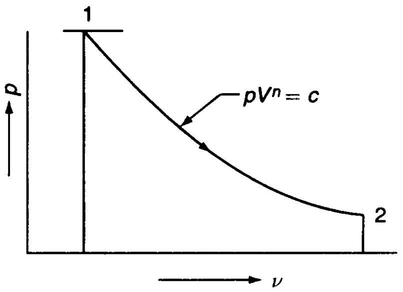


Fig. Ex. 9.12

$$pV^n = C$$
$$p_1 V_1^n = p_2 V_2^n$$

Taking logarithms and arranging the terms

$$n = \frac{\log \dfrac{p_1}{p_2}}{\log \dfrac{V_2}{V_1}}$$

Now, at 0.7 MPa, 200°C, from Tables A.2

$$v_1 = 0.352\ \text{m}^3/\text{kg}$$
$$h_1 = 2850.1\ \text{kJ/kg}$$

$\therefore$ Total volume, V_1, at state 1 = $0.352 \times 0.02 = 0.00704\ \text{m}^3$

Displaced volume = $\frac{\pi}{4} d^2 \cdot l = \frac{\pi}{4} \times (0.28)^2 \times 0.305 = 0.0188\ \text{m}^3$

∴ Total volume V_2 after expansion $= 0.0188 + 0.00704 = 0.02584\ \text{m}^3$

$$n = \frac{\log \dfrac{0.6}{0.12}}{\log \dfrac{0.02584}{0.00704}} = \frac{\log 5}{\log 3.68} = 1.24 \qquad \textit{Ans.}\ \text{(a)}$$

Work done by steam in the expansion process

$$W_{1-2} = \int_{V_1}^{V_2} p\mathrm{d}V = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{6 \times 10^5\ \text{N/m}^2 \times 0.00704\ \text{m}^2 - 1.2 \times 10^5\ \text{N/m}^2 \times 0.02584\ \text{m}^3}{1.24 - 1}$$

$$= \frac{4224 - 3100.8}{0.24}\ \text{Nm} = 4680\ \text{Nm} = 4.68\ \text{kJ} \qquad \textit{Ans.}\ \text{(b)}$$

Now $V_2 = 0.02584\ \text{m}^3$

∴ $v_2 = \dfrac{0.02584}{0.02} = 1.292\ \text{m}^3\text{/kg}$

Again $v_2 = v_{\text{f}0.12\text{MPa}} + x_2 v_{\text{fg}0.12\text{MPa}}$

or $1.292 = 0.0010476 + x_2 \times 1.4271$

∴ $x_2 = \dfrac{1.291}{1.427} = 0.906$

At 0.12 MPa, $u_\text{f} = 439.3\ \text{kJ/kg}, \quad u_\text{g} = 2512.0\ \text{kJ/kg}$

∴ $u_2 = 439.3 + 0.906\,(2512 - 439.3) = 2314.3\ \text{kJ/kg}$

Again $h_1 = 2850.1\ \text{kJ/kg}$

∴ $u_1 = h_1 - p_1v_1 = 2850.1 - \dfrac{0.6 \times 10^6 \times 0.00704 \times 10^{-3}}{0.02}$

$= 2850.1 - 211.2 = 2638.9\ \text{kJ/kg}$

By the first law

$$Q_{1-2} = U_2 - U_1 + W_{1-2} = m\,(u_2 - u_1) + W_{1-2}$$

$$= 0.02\,(2314.3 - 2638.5) + 4.68$$

$$= -\,6.484 + 4.68 = -\,1.804\ \text{kJ} \qquad \textit{Ans.}\ \text{(c)}$$

Example 9.13

A large insulated vessel is divided into two chambers, one containing 5 *kg of dry saturated steam at* 0.2 *MPa and the other* 10 *kg of steam,* 0.8 *quality at* 0.5 *MPa. If the partition between the chambers is removed and the steam is mixed thoroughly and allowed to settle, find the final pressure, steam quality, and entropy change in the process.*

Solution The vessel is divided into chambers, as shown in Fig. Ex. 9.13.1.

At 0.2 MPa, $v_\text{g} = v_1 = 0.8857\ \text{m}^3\text{/kg}$

∴ $V_1 = m_1v_1 = 5 \times 0.8857 = 4.4285\ \text{m}^3$

At 0.5 MPa, $v_2 = v_f + x_2 v_{fg} = 0.001093 + 0.8 \times 0.3749 = 0.30101$ m³/kg

$\therefore$ $V_2 = m_2\, v_2 = 10 \times 0.30101 = 3.0101$ m³

$\therefore$ Total volume, $V_m = V_1 + V_2 = 7.4386$ m³ (of mixture)

Total mass of mixture, $m_m = m_1 + m_2 = 5 + 10 = 15$ kg

$\therefore$ Specific volume of mixture

$$v_m = \frac{V_m}{m_m} = \frac{7.4386}{15} = 0.496 \text{ m}^3/\text{kg}$$

By energy balance

$$m_1\, u_1 + m_2\, u_2 = m_3\, u_3$$

At 0.2 MPa, $h_g = h_1 = 2706.7$ kJ/kg

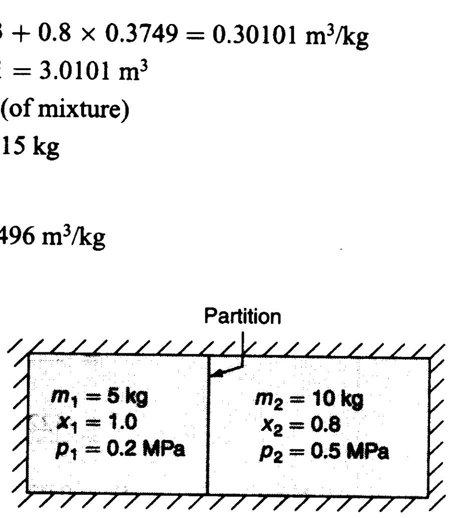
$u_1 = h_1 - p_1 v_1 \cong 2706.7$ kJ/kg

At 0.5 MPa, $h_2 = h_f + x_2 h_{fg} = 640.23 + 0.8 \times 2108.5 = 2327.03$ kJ/kg

$u_2 = h_2 - p_2\, v_2 \cong h_2 = 2327.03$ kJ/kg

$$h_3 = h_m = \frac{5 \times 2706.7 + 10 \times 2327.03}{15}$$

$\therefore$ $= 2453.6$ kJ/kg $\cong u_3$



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Now for the mixture

$h_3 = 2453.6$ kJ/kg $= u_3$

$v_3 = 0.496$ m³/kg

From the Mollier diagram, with the given values of h and v, point 3 after mixing is fixed (Fig. Ex. 9.13.2).

$x_3 = 0.870$ *Ans.*

$s_3 = 6.29$ kJ/kg K

$p_3 = 3.5$ bar *Ans.*

$s_4 = s_{g\,0.2\text{MPa}} = 7.1271$ kJ/kg K

$s_2 = s_{f\,0.5\text{MPa}} + 0.8 s_{fg\,0.5\text{MPa}}$

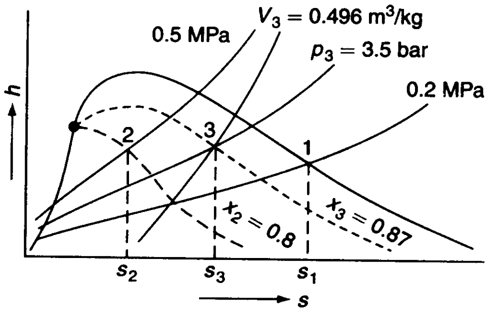
$= 1.8607 + 0.8 \times 4.9606$

– 5.8292 kJ/kg K

Entropy change during the process

$= m_3\, s_3 - (m_1\, s_1 + m_2\, s_2)$

$= 15 \times 6.298 - (5 \times 7.1271 + 10 \times 5.8292) = 0.43$ kJ/K *Ans.*



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Example 9.14

Steam generated at a pressure of 6 *MPa and a temperature of* 400°C *is supplied to a turbine via a throttle valve which reduces the pressure to* 5 *MPa. Expansion in the turbine is adiabatic to a pressure of* 0.2 *MPa, the isentropic efficiency (actual enthalpy drop/isentropic enthalpy drop) being* 82%. *The surroundings are at* 0.1 *MPa,* 20°C. *Determine the availability of steam before and after the throttle valve and at the turbine exhaust, and calculate the specific work output from the turbine. The K.E. and P.E. changes are negligible.*

Solution Steady flow availability Ψ is given by

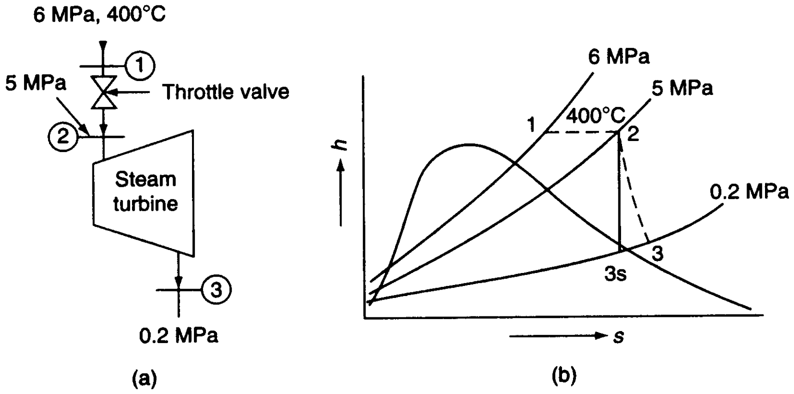
$$\Psi = (h - h_0) - T_0 (s - s_0) + \frac{1}{2} V_1^2 + g(Z - Z_0)$$

where subscript 0 refers to the surroundings. Since the K.E. and P.E. changes are negligible

$$\Psi_1 = \text{Availability of steam before throttling}$$
$$= (h_1 - h_0) - T_0(s_1 - s_0)$$

At 6 MPa, 400°C (Fig. Ex. 9.14)

$$h_1 = 3177.2 \text{ kJ/kg}$$
$$s_1 = 6.5408 \text{ kJ/kg K}$$



[illegible]

At 20°C

$$h_0 = 83.96 \text{ kJ/kg}$$
$$s_0 = 0.2966 \text{ kJ/kg K}$$

$$\therefore \quad \Psi_1 = (3177.2 - 83.96) - 293\,(6.5408 - 0.2966)$$
$$= 3093.24 - 1829.54 = 1263.7 \text{ kJ/kg}$$ *Ans.*

Now $h_1 = h_2$, for throttling

At $h = 3177.2$ kJ/kg and $p = 5$ MPa, from the superheated steam table

$$\left.\begin{aligned} t_2 &= 390°\text{C} \\ s_2 &= 6.63 \text{ kJ/kg K} \end{aligned}\right\} \text{by linear interpolation}$$

$$\therefore \quad \Psi_2 = \text{Availability of steam after throttling}$$
$$= (h_2 - h_0) - T_0\,(s_2 - s_0) = (3177.2 - 83.96) - 293\,(6.63 - 0.2966)$$
$$= 3093.24 - 1855.69 = 1237.55 \text{ kJ/kg}$$

Decrease in availability due to throttling

$$= \Psi_1 - \Psi_2 = 1263.7 - 1237.55 = 26.15 \text{ kJ/kg}$$

Now

$$s_2 = s_{3s} = 6.63 = 1.5301 + x_{3s}\,(7.1271 - 1.5301)$$

$$\therefore \quad x_{3s} = \frac{5.10}{5.5970} = 0.9112$$

$$h_{3s} = 504.7 + 0.9112 \times 2201.9 = 2511.07 \text{ kJ/kg}$$

$$h_2 - h_{3s} = 3177.2 - 2511.07 = 666.13 \text{ kJ/kg}$$

$$\therefore \quad h_2 - h_3 = h_{is}(h_1 - h_{3s}) = 0.82 \times 666.13 = 546.2 \text{ kJ/kg}$$

$$\therefore \quad h_3 = 2631 \text{ kJ/kg} = 504.7 + x_2 \times 2201.7$$

$$\therefore \quad x_3 = \frac{2126.3}{2201.7} = 0.966$$

$$s_3 = 1.5301 + 0.966 \times 5.597 = 6.9368$$

$$\therefore \quad \Psi_3 = \text{Availability of steam at turbine exhaust}$$

$$= (h_3 - h_0) - T_0(s_3 - S_0) = (2631 - 83.96) - 293\,(6.9368 - 0.2966)$$

$$= 2547.04 - 1945.58 = 601.46 \text{ kJ/kg}$$

Specific work output from the turbine

$$= h_2 - h_3 = 3177.2 - 2631 = 546.2 \text{ kJ/kg}$$ *Ans.*

The work done is less than the loss of availability of steam between states 2 and 3, because of the irreversibility accounted for by the isentropic efficiency.

Example 9.15

A steam turbine receives 600 *kg/h of steam at* 25 *bar,* 350°*C. At a certain stage of the turbine, steam at the rate of* 150 *kg/h is extracted at* 3 *bar,* 200°*C. The remaining steam leaves the turbine at* 0.2 *bar,* 0.92 *dry. During the expansion process, there is heat transfer from the turbine to the surroundings at the rate of* 10 *kJ/s. Evaluate per kg of steam entering the turbine* (*a*) *the availability of steam entering and leaving the turbine,* (*b*) *the maximum work, and* (*c*) *the irreversibility. The atmosphere is at* 30°*C.*

Solution At 25 bar, 350°C

$$h_1 = 3125.87 \text{ kJ/kg}$$

$$s_1 = 6.8481 \text{ kJ/kg K}$$

At 30°C,

$$h_0 = 125.79 \text{ kJ/kg}$$

$$s_0 = s_{f\,30^\circ C} = 0.4369 \text{ kJ/kg K}$$

At 3 bar, 200°C

$$h_2 = 2865.5 \text{ kJ/kg}$$

$$s_2 = 7.3115 \text{ kJ/kg K}$$

At 0.2 bar (0.92 dry)

$$h_f = 251.4 \text{ kJ/kg}$$

$$h_{fg} = -2358.3 \text{ kJ/kg}$$

$$s_f = 0.8320 \text{ kJ/kg K}$$

$$s_g = 7.9085 \text{ kJ/kg K}$$

$$\therefore \quad h_3 = 251.4 + 0.92 \times 2358.3 = 2421.04 \text{ kJ/kg}$$

$$s_3 = 0.8320 + 0.92 \times 7.0765 = 7.3424 \text{ kJ/kg K}$$

The states of steam are shown in Fig. Ex. 9.15.

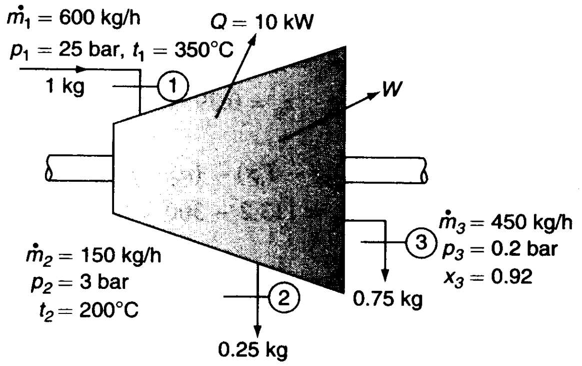


Fig. Ex. 9.15

(a) Availability of steam entering the turbine

$$\Psi_1 = (h_1 - h_0) - T_0\,(s_1 - s_0)$$
$$= (3125.87 - 125.79) - 303\,(6.8481 - 0.4369)$$
$$= 3000.08 - 1942.60 = 1057.48 \text{ kJ/kg} \qquad \textit{Ans.}$$

Availability of steam leaving the turbine at state 2,

$$\Psi_2 = (h_2 - h_0) - T_0(s_2 - s_0) = (2865.5 - 125.79) - 303\,(7.3115 - 0.4369)$$
$$= 2739.71 - 2083.00 = 656.71 \text{ kJ/kg} \qquad \textit{Ans.}$$

Availability of steam leaving the turbine at state 3,

$$\Psi_3 = (h_3 - h_0) - T_0\,(s_3 - s_0)$$
$$= (2421.04 - 125.79) - 303\,(7.3524 - 0.4369)$$
$$= 199.85 \text{ kJ/kg}$$

(b) Maximum work per kg of steam entering the turbine

$$W_{\text{rev}} = \Psi_1 - \frac{m_2}{m_1}\,\Psi_2 - \frac{m_3}{m_1}\,\Psi_3 = 1057.48 - 0.25 \times 656.71 - 0.75 \times 199.85$$
$$= 743.41 \text{ kJ/kg} \qquad \textit{Ans.}$$

(c) Irreversibility

$$I = T_0(w_2\,s_2 + w_3\,s_3 - w_1\,s_1) - Q$$
$$= 303\,(150 \times 7.3115 + 450 \times 7.3424 - 600 \times 6.8481) - (-10 \times 3600)$$
$$= 303\,(1096.73 + 3304.08 - 4108.86) + 36000 = 124{,}460.85 \text{ kJ/h}$$
$$= 124.461 \text{ MJ/h} = \frac{124.461 \times 103}{600} = 207.44 \text{ kJ/kg} \qquad \textit{Ans.}$$

Example 9.16

Determine the exergy of (a) 3 kg of water at 1 bar and 90°C, (b) 0.2 kg of steam at 4 MPa, 500°C and (c) 0.4 kg of wet steam at 0.1 bar and 0.85 quality, (d) 3 kg of ice at 1 bar – 10°C. Assume a dead state of 1 bar and 300 K.

Solution At the dead state of 1 bar, 300 K,

$$u_0 = 113.1 \text{ kJ/kg}, \quad h_0 = 113.2 \text{ kJ/kg K.}$$
$$v_0 = 0.001005 \text{ m}^3\text{/kg}, \quad s_0 = 0.395 \text{ kJ/kg}$$

Exergy of the system:

$$\phi = m[(u + p_0 v - T_0 s) - (u_0 + p_0 v_0 - T_0 s_0)]$$

Now, $u_0 + p_0 v_0 - T_0 s_0 = h_0 - T_0 s_0 = 113.2 - 300 \times 0.395 = -5.3$ kJ/kg

(a) For water at 1 bar, 90°C

$$u = 376.9 \text{ kJ/kg}, \quad h = 377 \text{ kJ/kg}, \quad v = 0.001035 \text{ m}^3\text{/kg}$$
$$s = 1.193 \text{ kJ/kg K.}$$

Since $p = p_0$,

$$u + p_0 v - T_0 s = u + pv - T_0 s = h - T_0 s$$
$$= 377 - 300 \times 1.193 = 19.1 \text{ kJ/kg}$$

Hence, $\phi = 3[19.1 - (-5.3)] = 3 \times 24.4 = 73.2$ kJ *Ans.*

(b) At $p = 4$ MPa, $t = 500$°C

$$u = 3099.8, \quad h = 3446.3 \text{ kJ/kg}, \quad v = 0.08637 \text{ m}^3\text{/kg}$$
$$s = 7.090 \text{ kJ/kg K}$$
$$u + p_0 v - T_0 s = 3099.8 + 100 \times 0.08637 - 300 \times 7.090 = 981.4 \text{ kJ/kg}$$

$\phi = 0.2\,[981.4 - (-5.3)] = 197.34$ kJ *Ans.*

(c) At 0.1 bar, 0.85 quality,

$$u = 192 + 0.85 \times 2245 = 2100.25 \text{ kJ/kg}$$
$$h = 192 + 0.85 \times 2392 = 2225.2 \text{ kJ/kg}$$
$$s = 0.649 + 0.85 \times 7.499 = 7.023 \text{ kJ/kg K}$$
$$v = 0.001010 + 0.85 \times 14.67 = 12.47 \text{ m}^3\text{/kg}$$
$$u + p_0 v - T_0 s = 2100.25 + 100 \times 12.47 - 300 \times 7.023 = 1240.4 \text{ kJ/kg}$$

$\phi = 0.4\,[1240.4 - (-5.3)] = 498.3$ kJ *Ans.*

(d) Since $p = p_0$,

$$\phi = U - U_0 + p_0(V - V_0) - T_0(S - S_0) = H - H_0 - V(p - p_0) - T_0(S - S_0)$$
$$= m[(h - h_0) - T_0(s - s_0)]$$

At 100 kPa, −10°C,

$$h = -354.1 \text{ kJ/kg and } s = -1.298 \text{ kJ/kg K}$$

$\phi = 3\,[-354.1 - 113.2 - 300\,(-1.298 - 0.0395)] = 81.2$ kJ. *Ans.*

Example 9.17

A flow of hot water at 90°C is used to heat relatively cold water at 25°C to a temperature of 50°C in a heat exchanger. The cold water flows at the rate of 1 kg/s. When the heat exchanger is operated in the parallel mode, the exit temperature of the hot water stream must not be less than 60°C. In the counterflow operation, the exit temperature of hot water can be as low as 35°C. Compare the second law efficiency and the rate of exergy destruction in the two modes of operation. Take $T_0 = 300$ K.

Solution Given: $t_{h_1} = 90°C$, $t_{c_1} = 25°C$, $t_{c_2} = 60°C$,

$$\dot{m}_c = 1 \text{ kg/s}, \quad T_0 = 300 \text{ K}.$$

The two modes of operation of (a) parallel flow and (b) counterflow are shown in Fig. Ex. 9.17.

In parallel flow mode (a), $t_{h_2} = 60°C$. Neglecting any heat loss,

$$\dot{m}_h c_h(t_{h_1} - t_{h_2}) = \dot{m}_c c_c(t_{c_2} - t_{c_1})$$

$$\dot{m}_h (90 - 60) = 1(50 - 25)$$

$$\dot{m}_h = 0.833 \text{ kg/s}$$

In counterflow mode, $t_{h_2} = 35°C$,

$$\dot{m}_h (90 - 35) = 1(50 - 25)$$

$$\dot{m}_h = \frac{25}{55} = 0.454 \text{ kg/s}$$

Thus, the counterflow arrangement uses significantly less hot water.

Assuming that the hot water stream on exit from the heat exchanger is simply dumped into the drain, the exergy flow rate of the hot water stream at entry is considered as the exergy input rate to the process.

$$a_{f1} = \dot{m}_h [(h_1 - h_0) - T_0(s_1 - s_0)]$$

At 300 K or 27°C, $h_0 = 113.2$ kJ/kg and $s_0 = 0.395$ kJ/kg K

At 90°C, $h_1 = 376.92$ kJ/kg, $s_1 = 1.1925$ kJ/kgK

$$a_{f_1} = 0.833 [(376.92 - 113.2) - 300 (1.1925 - 0.395)]$$

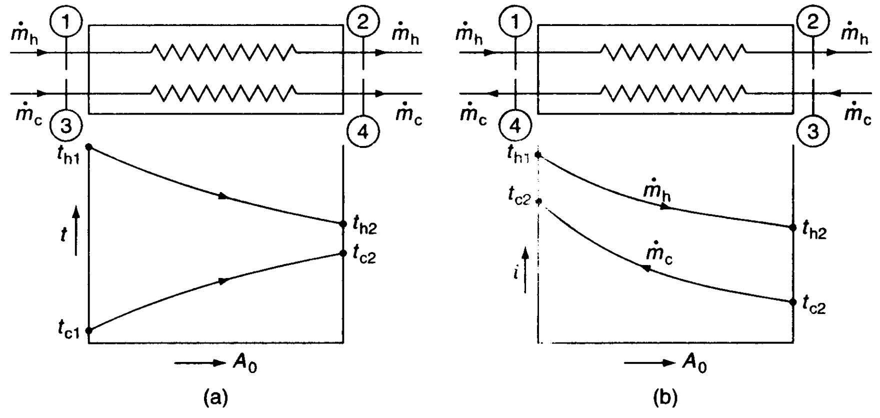
$$= 0.833 (263.72 - 239.25) = 20.38 \text{ kW}$$

Parallel flow:

At 60°C, $h_2 = 251.13$ kJ/kg, $s_2 = 0.8312$ kJ/kg K

At 25°C, $h_3 = 104.89$ kJ/kg, $s_3 = 0.3674$ kJ/kg K

At 50°C, $h_4 = 209.33$ kJ/kg, $s_4 = 0.7038$ kJ/kg K



(a) (b)

Fig. Ex. 9.17

Rate of exergy gain:

$$= \dot{m}_c [(h_4 - h_3) - T_0(s_4 - s_3)]$$

$$= 1 [(209.33 - 104.89) - 300(0.7038 - 0.3674)]$$

$$= 104.44 - 100.92 = 3.52 \text{ kW}$$

$$(\eta_{II})_P = \frac{3.52}{20.38} = 0.172 \text{ or } 17.2\%$$

Rate of exergy loss by hot water:

$$= \dot{m}_h [(h_1 - h_2) - T_0 (s_1 - s_2)]$$

$$= 0.833 [(376.92 - 251.13) - 300 (1.1925 - 0.8312)]$$

$$= 0.833 (125.79 - 108.39) = 14.494 \text{ kW}$$

Rate of irreversibility or exergy destruction:

$$= 14.494 - 3.52 = 10.974 \text{ kW}$$

If the hot water stream is not dumped to the drain,

$$\eta_{II, P} = \frac{3.52}{14.494} = 0.243 \quad \text{or} \quad 24.3\%$$ *Ans.*

Counterflow:

At 35°C, $h_2 = 146.68$ kJ/kg, $s_2 = 0.5053$ kJ/kg K

Rate of exergy gain of cold water $= \dot{m}_h [(h_4 - h_3) - T_0(s_4 - s_3)] = 3.52$ kW

(same as in parallel flow)

Rate of exergy input (if existing hot water is dumped to the surroundings)

$$= 0.454 (263.72 - 239.25) = 11.11 \text{ kW}$$

$$\eta_{II, C} = \frac{3.52}{11.11} = 0.3168 \text{ or } 31.68\%$$

Rate of exergy loss of hot water:

$$= \dot{m}_h [(h_1 - h_2) - T_0(s_1 - s_2)]$$

$$= 0.454 [(376.92 - 146.68) - 300 (1.1925 - 0.5053)]$$

$$= 0.454 (230.24 - 206.16) = 10.94 \text{ kW}$$

$$\eta_{II, C} = \frac{3.52}{10.94} = 0.3217 \quad \text{or} \quad 32.17\%$$ *Ans.*

Rate of irreversibility or exergy destruction:

$$= 10.94 - 3.52 = 7.42 \text{ kW}$$ *Ans.*

The second law efficiency for the counterflow arrangement is significantly higher and the rate of irreversibility is substantially lower compared to the parallel flow arrangement.

Example 9.18

A small geothermal well in a remote desert area produces 50 kg/h of saturated steam vapour at 150°C. The environment temperature is 45°C. This geothermal steam will be suitably used to produce cooling for homes at 23°C. The steam will emerge from this system as saturated liquid at 1 atm. Estimate the maximum cooling rate that could be provided by such a system.